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HYDRODYNAMICS OF LIQUID HELIUM II*

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This note gives a brief account of a macroscopic hydrodynamic theory of liquid helium II which can describe most of the experimental observations on the thermal and mechanical behavior of liquid helium below the λ-point. The theory evolves from a one-fluid concept of liquid helium in which a long-range order exists in the momentum space. The transfer of long-range order (disorder) is then the superconductivity of heat. By assuming that one is dealing with a classical thermomechanical medium, it can be mathematically proved that there is a momentum transfer associated with the reversible energy transfer even when the momentum of the fluid is zero. This is a typical feature of the usual two-fluid theory; indeed, a two-fluid model can be constructed, in a purely mathematical manner, out of the present one-fluid theory. The basic equations, one governing the rate of change of momentum and the other governing the rate of change of the reversible heat flux vector, can then be reduced to the ordinary equations of Landau. There is, however, no longer any compelling reason to believe that the motion of the superfluid component must be irrotational. (See Lee and Yang.[1])

Accordingly, we propose to abandon the often-used basic assumption that the superfluid component of the flow is irrotational. (See Lifsič and Halatnikov.[2]) We next include certain dissipative terms in our equations of motion dependent on the gradients of physical quantities, in agreement with Lifsič and Halatnikov. Since the equations of motion now contain the second-order space derivatives of the superfluid component, we need a boundary condition for the tangential component of the superfluid velocity. Bearing in mind the requirement of covariance with respect to coordinate transformations, we propose the following general form for the boundary condition:

$$\partial \vec{v}_{s,T}/\partial N = f(\vec{q}^2)\vec{q}, \tag{1}$$

where $\vec{q}$ is the tangential component of the difference between superfluid velocity and wall velocity, and $\partial \vec{v}_{s,T}/\partial N$ is the tangential component of the normal derivative of the superfluid velocity. Before a molecular theory can be developed, the form of $f(\vec{q}^2)$ must be determined by experiments. For small velocities, we assume

$$f(\vec{q}^2) = \alpha + \beta\vec{q}^2 + \cdots . \tag{2}$$

From Andronikashvili's disk pile experiment, we know that α must be negligibly small. Thus, the simplest form for $f(\vec{q}^2)$ is

$$f(\vec{q}^2) = \beta\vec{q}^2. \tag{3}$$

With this simple assumption, we can derive the following results for experiments involving simple geometry.

(A) For rotating bucket experiments, the theory predicts uniform rotation of the superfluid component, but at a speed lower than that of the normal component, because of the boundary condition (1). The ratio of angular momentum L actually acquired by helium II to the ratio

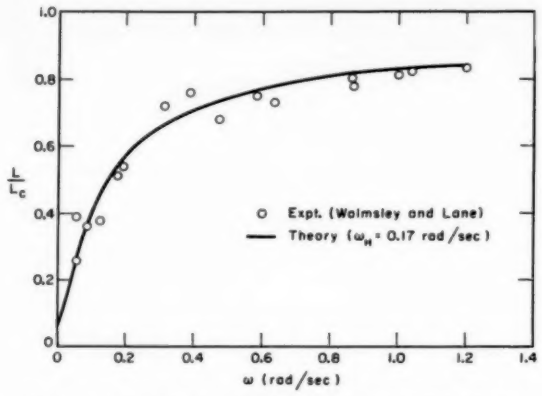

FIG. 1. Angular momentum of liquid helium II in a rotating bucket.

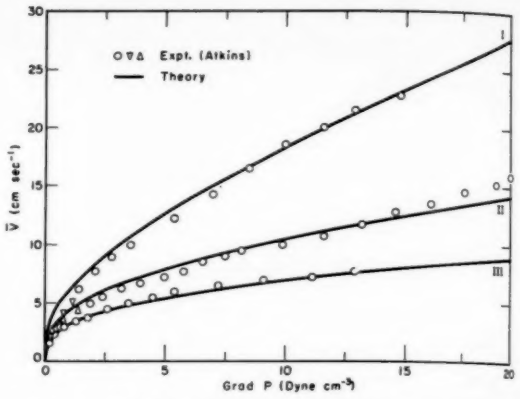

FIG. 2. Velocity of flow of liquid helium II through tubes of various diameters. (I) $d = 4.40\times10^{-2}$ cm (∇ for short tube). (II) $d = 2.03\times10^{-2}$ cm (Δ for short tube). (III) $d = 0.815\times10^{-2}$ cm.

L_c for ordinary fluids is given by

$$L/L_c = s + x(1-s),$$

$$\omega/\omega_H = \tfrac{1}{2}[s/(1-s)^3]^{1/2}, \quad (4)$$

where x is the fraction of normal fluid present, ω is the angular velocity of the bucket, and ω_H is related to the molecular parameter β by

$$\omega_H^2 = 4\beta a^{-3}. \quad (5)$$

In this formula, a is the radius of the bucket.

(B) For isothermal flow through a tube (or channel), the amount of flow depends on the pressure gradient Γ in the following manner:

$$\text{Mean velocity } \bar{v} = Ad^2\Gamma + Bd^{1/3}\Gamma^{1/3}, \quad (6)$$

where A and B are molecular parameters, and d is the diameter of the tube (or width of the channel).

In Fig. 1, one set of experimental results of Walmsley and Lane[3] is compared with Eq. (4). There is only one other set of data at the same temperature which is, however, not sufficiently accurate to test the validity of (5). In Fig. 2, the experimental results of Atkins[4] are compared with (6). One may notice that the theoretical and experimental results agree for tubes of various sizes.

Preliminary analyses also show agreement of the theory with experimental results for fountain effect and for damping experiments at finite amplitudes.[5] These will be discussed in a more complete paper. It also follows from the present concepts that second sound should propagate through rotating liquid helium II in essentially the same manner as in the nonrotating case—a fact observed by Hall and Vinen.[6] The detailed examination of attenuation of second sound in rotating helium is left for further investigation; preliminary considerations also show general agreement with their results.

I should like to express my thanks to Professor C. N. Yang for many discussions during the development of the one-fluid concept of liquid helium II while both of us were at Madison during the summer of 1958. I wish also to thank Professor C. T. Lane for private communications and comments on the data shown in Fig. 1, and to thank Mr. Alfred Clark, Jr., for his help in the numerical analysis of the experimental data shown in Fig. 2. Thanks are also due to my colleagues L. Tisza, Kerson Huang, and Charles E. Chase for their interest and discussions.

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MAGNETIC SUSCEPTIBILITY OF PHOTO-GENERATED CURRENT CARRIERS IN GERMANIUM

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The experiments described in this note relate to measurements of the magnetic susceptibility, χ, of free carriers in germanium by means of a method fundamentally different from those heretofore employed. We observe a carrier χ several hundred times bigger than that expected in the light of previous work reported in the recent literature.[1] In the past, the carrier χ has been derived by subtracting the χ measured for a pure specimen from the χ of a heavily doped one. In another approach, the temperature of an intrinsic specimen has been raised so as to introduce many pairs in equilibrium. The carrier χ was then obtained by subtracting the low-temperature, or "carrier-free" χ from the total high-temperature χ.[2]

The results of these experiments fit fairly well the expectations derived from the band structure. Nevertheless, the validity of the subtractions is open to some general doubts.[3]

The experiments reported here are based on a new experimental technique which does not entail these difficulties—although it is not entirely free from others. Here the change in magnetism of the crystal due to the introduction of hole electron pairs injected by an external light source is measured directly. It is difficult to generate more than $\sim 10^{15}$ carriers/cm^3 by means of reasonable light intensities. Therefore, if one assumes the Landau-Peierls diamagnetism, one requires a detection sensitivity of at least 10^{-11} mass susceptibility units at room temperatures, which is ~100 times the sensitivity of the high-quality apparatus usually employed for measuring the χ of solids. This sensitivity was in fact achieved in the present arrangement. The method employed is basically the Faraday method, in which a small specimen is suspended in an inhomogeneous magnetic field. Our apparatus measures only the alternating component of the force produced when the sample is illuminated by light, modulated at the detection frequency of the apparatus (16 cps). The measurements are nevertheless "dc" since all pertinent processes are faster than the detection frequency. However, "dc" drifts in the total χ do not interfere with the observations. The detector is synchronous with the light modulation.

The specimen used for all the measurements reported here was nearly intrinsic Ge (35 ohm-cm, n-type, at 300°K). The low-level minority carrier mean life was 500 μsec. The dimensions were $2\times1\times0.2$ cm. For most of the measurements the specimen was etched so as to give low surface recombination velocity, s. A specimen adjacent (in the parent crystal) to the one used for the χ runs was used for observations of the photoconductivity. The photon flux and injected carrier density were determined by a combination of photoconductive decay lifetime measurements and steady-state photoconductivity.

The dependence of the photoconductivity on light intensity was linear at room temperature, up to an absorbed photon flux of 3×10^{16}/cm^2 sec. At higher photon fluxes, a square root dependence set in, implying a $(\Delta n)^{-1}$ dependence of the mean life, τ. The dependence of the carrier density on the light level may easily be understood in terms of the Shockley-Read theory applied to an intrinsic semiconductor.[4]

Hole-electron pairs generated by light may give rise to two effects. The first is the diamagnetic susceptibility, χ_d, of the added carriers; the other is a macroscopic loop current which produces a magnetic moment. Although loop currents could be eliminated through the use of pulverized specimens, none were employed because of the difficulty in eliminating surface effects and estimating the correct photocarrier density.

To obtain the magnetic susceptibility, therefore, it is necessary to separate and remove that part of the force signal due to macroscopic moments from the total signal. The origin of the macroscopic moments is the photoelectromagnetic (PEM) effect which may arise when there is a concentration gradient of carrier pairs in the presence of a magnetic field. The PEM short-circuit current, i_s, is proportional to $\vec{v}_d\times\vec{H}$, where $\vec{v}_d$ is the diffusion current associated with the concentration gradient. When no electrical connections are made to the specimen, i_s flows within it. This circulating current gives rise to a magnetic moment, $\vec{\mu}_{\mathrm{PEM}}\propto\vec{v}_d\times\vec{H}$, since the current does not cancel locally but only when summed over the profile of the entire specimen.[5] The $\vec{\mu}_{\mathrm{PEM}}$ results in an observable force proportional to $\vec{\mu}_{\mathrm{PEM}}\cdot\mathrm{grad}H$. The effect has been predicted by Aigrain and Garreta[6] and implied by

Frenkel.[7]

When $\vec{v}_d$ points into the sample, the force is paramagnetic in sign. When $\vec{v}_d$ points from the interior outward, it is diamagnetic. The latter condition may be achieved either by treating the surfaces in such a way that the carrier recombination is higher there than in the sample interior, or by employing nonuniform illumination so that a component of $\vec{v}_d$ is perpendicular to $\vec{H}$. The diamagnetic effect may occur when the primary carrier diffusion is along $\vec{H}$ and thus does not produce a normal PEM effect. The PEM effects were exhaustively studied, both for their intrinsic value and so that a correct separation between them and the susceptibility could be performed. The short-circuit PEM current, i_s, which is proportional to $\vec{\mu}_{\text{PEM}}$, was also measured in the experimental geometry in order to permit comparison of its characteristics with the information from the force measurements.

We found that if a specimen with etched, low surface-recombination-velocity surfaces is suspended in the inhomogeneous field and illuminated with very uniform light, two types of forces are seen. If $|\vec{v}_d \times \vec{H}| = v_d H$ (90° orientation), a paramagnetic signal is observed, whereas when $|\vec{v}_d \times \vec{H}| = 0$ (0° orientation), the signal is diamagnetic. Both signals depend on the temperature, T, the paramagnetic signal rising and the diamagnetic signal falling with increasing T. It was possible to resolve the 90° paramagnetic signal into two components having opposite signs. The paramagnetic component exhibited the characteristic PEM drop-off of quantum efficiency at long light wavelengths, as well as the same light intensity and (small) temperature dependence of the measured PEM i_s. As the incident light becomes more penetrating at long wavelengths, the quantum efficiency of the normal PEM effect decreases sooner than that of the photoconductivity due to the reduction of the carrier concentration gradient. The second component of the force signal was diamagnetic and depended on T, light intensity, and wavelength in exactly the same way as the 0° effect. Thus the total general force signal was found to consist of one component due to the normal PEM effect, and a second one which was invariant to the relative orientation of incident light (or $\vec{v}_d$) with respect to $\vec{H}$. Since a $\vec{\mu}_{\text{PEM}}$ can be generated in the 0° orientation only with nonuniform light and cannot remain invariant under a rotation of the sample through 90°, and since the observed T dependence of the diamagnetic signal is much stronger than that of any PEM effect, we conclude that the diamagnetic signal originates in light-induced changes of the susceptibility, χ_d. In the 90° orientation the PEM and susceptibility signals are approximately equal at 300°K and with a photon flux of 1.3×10^{18} photons/cm² sec, whereas in the 0° orientation only the susceptibility signal is observed. We have also shown, by means of a blackened specimen, that the diamagnetic signal is neither a manifestation of the Nernst effect, nor of synchronous temperature-induced fluctuations of the total χ.

The outstanding characteristic of χ_d is its magnitude; it is 400 times larger than the χ_d expected from summing the contributions of the photogenerated free carriers, assuming the Landau-Peierls theory and the cyclotron resonance effective masses averaged in the usual way.[8] The χ_d measured at 300°K for an incident photon flux $I = 1.3 \times 10^{18}$/cm² sec, resulting in 7×10^{14} pairs/cm³, is 1.02×10^{-9} mass susceptibility units. Of equally fundamental interest is the dependence of χ_d on I. Over the temperature range from 300°K to 420°K, χ_d is linear in I, whereas the light-generated pair concentration, $\frac{1}{2}(\Delta n + \Delta p)$, varies as $I^{1/2}$ at 300°K and approaches linearity at the higher temperatures. In these experiments $2 \times 10^{16} \leq I \leq 10^{18}$ photons/cm² sec. Furthermore, χ_d drops quite rapidly as the temperature increases, although $\Delta n + \Delta p$ does not. According to a preliminary empirical equation which fits all the data obtained so far, the drop-off in χ_d, for T in the vicinity of 400°K, is given by $\chi_d = \text{const} \times I \times (0.0025 \text{ ev}/KT)(1 - 300/T)^{-1}$. This rather suggestive relation has not yet been tested at low T. The wavelength dependence of χ_d is the same as that of the photoconductivity. It should be pointed out here that although the measured susceptibilities reported in this paper are anomalously large, they do not in fact directly contradict any previous measurements. The conflict occurs only with the theoretical extrapolation of those measurements to the case of carrier pairs at our concentrations and temperatures.

It is clear that χ_d and $\Delta n + \Delta p$ are not linearly related. In the low-temperature range, where $\Delta n + \Delta p \gg n_0 + p_0$, the equilibrium carrier density, χ_d, is apparently proportional to $(\Delta n + \Delta p)^2$, whereas at the high T, where $\Delta n + \Delta p \cong n_0 + p_0$, linear dependence is approached. It is therefore not unreasonable to suppose that the observed

anomalously large χ_d is due to some correlation or exciton effect, which would be expected to be proportional to a quadratic form in Δn, Δp, n_0, and p_0.

There exists the rather remote possibility that the light affects the large paramagnetic component of the lattice susceptibility directly. However, all processes which do not in some way relate to correlation or to another breakdown in the usual theory may almost certainly be ruled out by the magnitude of the observed χ_d. Further experiments, using extrinsic samples, together with measurements at lower temperatures, are expected to yield the answers to many of these questions.

We wish to acknowledge the help of B. Tompkins in the construction of the apparatus, as well as several very helpful conversations with M. A. Lampert, J. Loferski, and A. Rose.

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PIEZOELECTRIC PRODUCTION OF MICROWAVE PHONONS

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Recently Baranskii,[1] and Bömmel and Dransfeld[2] have reported the production and propagation of ultrasonic waves in crystalline quartz, excited by electromagnetic fields, in the frequency range of 10^8 to 2.5×10^9 cps.

We have been able to extend the frequency, at which ultrasonic wave propagation is detectable, to the vicinity of 10 000 Mc/sec. Specifically, we have successfully propagated longitudinal elastic waves at low temperatures in quartz bars, with optically flat ends and of various dimensions, placed between two re-entrant microwave cavities tuned to the transmitter frequency.

Figure 1 shows an oscilloscope trace of the following experiment carried out in a liquid helium bath. A pulse of microwave energy at 9370 Mc/sec is fed to one cavity, whereupon it is piezoelectrically converted to an elastic wave of the same frequency which travels along the quartz rod at the characteristic sonic velocity. Upon arrival of the wave at the receiver cavity, a part of its energy is reconverted to a radio-frequency signal and is detected and displayed on the oscilloscope, appearing as the first echo pulse. The remaining sound wave energy is reflected at the receiving end of the rod, traveling back to the transmitting end whereupon it is again reflected to the receiver cavity to produce another echo pulse, and so on, until the ultrasonic wave train dies out. It will be observed that, whereas the first echo pulse is detected at a time τ after the direct transmitter pulse, the second and all subsequent pulses are separated by a transit time of 2τ since these have made a

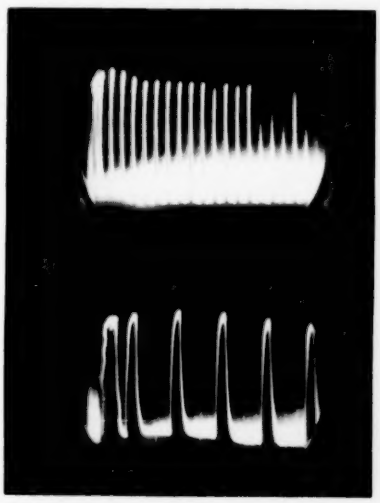

FIG. 1. Oscilloscope trace of the receiver output registering pulses of microwave power (9370 Mc/sec) acoustically delayed by a quartz rod 3 cm in length and submerged in liquid helium at 4.2°K. The lower trace is identical with the one above except for an expanded time base. Time increases from left to right. Note that the first echo appears at a time τ after the direct transmitter pulse (first pulse on photograph) whereas all subsequent echoes are spaced at intervals of 2τ (see text).

complete round trip. This feature is seen more clearly in the lower trace of Fig. 1, which exhibits the pulses on an expanded time base.

The dimensions of the quartz rod used in this experiment were 3 cm (length) by 0.3 cm (diameter), with the x axis along the length of the rod. The characteristic time (2τ) between pulses was about 11 microseconds, and the pulse duration, two microseconds. The peak rf power incident upon the transmitter cavity was 25 watts, derived from a 4J50 magnetron.

In Fig. 1, the first dozen or so pulses appear to have nearly the same amplitude due to saturation of the receiver. After correction for this effect, the amplitude of each successive pulse is as shown in Fig. 2, where the third echo pulse is normalized to 100. The pulse decay envelope is seen to be more complicated in detail than the expected simple exponential drop-off. The details of the decay curve appear to be a function of the microwave electric field configuration at the point of coupling in the transmitting cavity and of the particular quartz rod employed. Thus the irregular decay may be caused by partial coupling to other elastic modes of the rod in such a way that energy is alternately exchanged between them. This detail is under further investigation.

When the temperature is varied from 1.8°K to 20°K, it is found that the average attenuation increases somewhat more rapidly than linearly with temperature. At 77°K no ultrasonic wave propagation was observable. It is expected that umklapp processes will become effective in attenuating the wave in the temperature range between 20°K and 77°K.

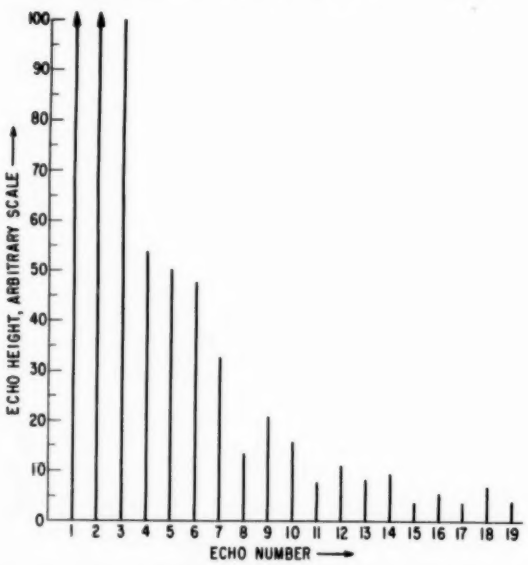

FIG. 2. Graph of relative echo amplitudes after correction for receiver saturation. Third echo pulse is normalized to 100.

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[2]H. E. Bommel and K. Dransfeld, Phys. Rev. Lett. 1, 234 (1958).

MAGNETIC OSCILLATIONS OF ULTRASONIC ATTENUATION IN A COPPER CRYSTAL AT LOW TEMPERATURES*

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In an earlier communication[1] we reported an apparently oscillatory dependence of ultrasonic attenuation in a polycrystalline sample of copper, this occurring at helium temperatures where the electron mean free path (l) was comparable to the ultrasonic wavelength (λ). The results were explained in terms of resonant conditions between the electron orbit diameter and the spatially periodic fields carried by the wave. It was shown that the average Fermi momentum calculated from such a picture was consistent with a value for copper of one free electron per atom. These measurements, however, were deficient in several obvious respects. Only one or two maxima and minima were discernible, and a polycrystal was used because at that time a single crystal of sufficiently long mean free path was not available. Furthermore, a subsequent theoretical analysis by Rodriguez[2] has suggested that such an oscillatory effect would not be expected (at least from a classical Boltzmann equation analysis), and so it was proposed that the observed effect was perhaps a consequence of the polycrystalline nature of the sample.

Here we report some more recent measurements made in a very pure copper single crystal which verify that a pronounced oscillatory effect is indeed found. Moreover, this effect is significantly anisotropic, suggesting that ultrasonic methods should be useful in obtaining information

about details of the electronic properties of metals.

Attenuation measurements were made of both longitudinal and shear waves at frequencies between 15 and 75 Mc/sec and in magnetic fields of various orientations at strengths up to 6500 gauss. The temperature was varied between 1.0° and 4.2°K but the results were found to be essentially temperature-independent, indicating that l was determined by impurity scattering. The attenuation was measured by observing a 1-μsec ultrasonic pulse reflected between two highly parallel faces of the sample, these being separated by about 1.25 cm. The faces of the samples were oriented with respect to the crystalline axes to an accuracy better than $\frac{1}{2}°$.

Figure 1 shows the results for a longitudinal wave of 75 Mc/sec propagated along the [001] direction. The magnetic field was applied in a plane normal to the propagation direction and could be rotated about this axis. Since the oscillatory effect should be periodic in H^{-1}, the results are plotted in Fig. 1 on a reciprocal scale. Figure 1(a) shows data for H along either a [100] or a [010] direction. About seven maxima and minima are found which are quite accurately periodic in H^{-1}. Only slightly different results are found for H within about ±15° of these directions. However, with H in the vicinity of the 45° directions, such as [110], the oscillations are changed considerably, a typical result being shown in Fig. 1(b).

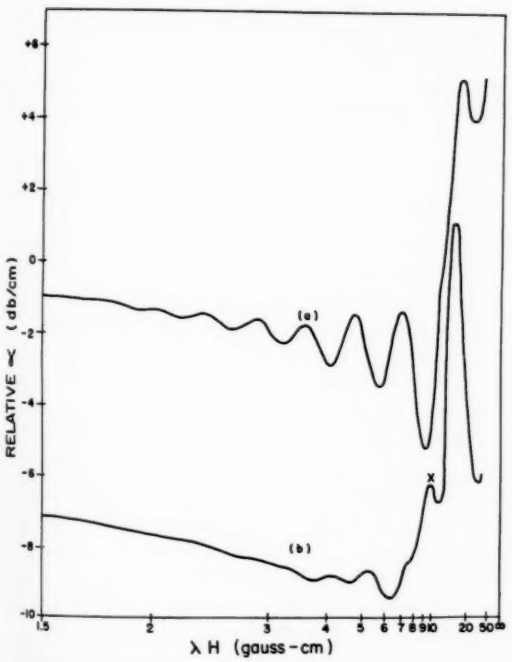

FIG. 1. Attenuation along [001] of 75-Mc/sec longitudinal wave vs λH plotted on reciprocal scale. Curve (a) is for H in the [100] direction, and (b) is for H in the [110] direction. Attenuation is measured relative to that for $H=0$, with the (b) curve arbitrarily displaced downward by 6 db/cm.

Although no satisfactory theory for such oscillatory behavior now exists, it is of interest to observe that the results are consistent with a model of the Fermi surface proposed by Pippard based on his observations of the anomalous skin effect.[3] In the ultrasonic effect one would expect the greatest contribution to come from those electrons in an extremal cross section normal to the direction of H. Pippard suggests that the Fermi surface in copper touches the zone boundary in the [111] directions, but is nearly spherical elsewhere. Thus for H along the principal axes the relevant electrons would lie on a nearly circular cross section of the Fermi surface. This is the orientation where the well-defined periodic effect is found. On the other hand, when H is along a 45° direction the relevant electrons are on a cross section that is broken up into four segments by the zone boundaries. Experimentally, it is in these directions that one sees a more complicated effect. In particular, another oscillation of a different period seems to enter. A maximum of this other period is designated by × in Fig. 1(b).

One can make an estimate of the Fermi momentum (p_f) from the measurements shown in Fig. 1. If it is assumed that between adjacent maxima the average orbit radius changes by $\lambda/2$, and that $\bar{r}=\bar{p}_\perp c/eH$, where $p_\perp$ is the momentum component perpendicular to H, then one finds $\bar{p}_\perp = 1.16\times10^{-19}$ g cm/sec for H along a principal axis. It seems likely that it is some average of $\bar{p}_\perp$ that matters and not the extremal value $p_\perp$. If the assumption of a spherical shell is used, then $\bar{p}_\perp = (\pi/4)p_f$, or $p_f = 1.47\times10^{-19}$ g cm/sec. According to Pippard's estimate, p_f in the extremal cross section perpendicular to [100] varies between 1.39×10^{-19} and 1.44×10^{-19} g cm/sec.

Contrary to the results in the polycrystal, the attenuation does not seem to approach zero as H becomes large (at least in all orientations). A similar effect was noted earlier in a tin single crystal.[4] The attenuation at high values of the field (i.e., for $\bar{r} < \lambda/2$) also is found to be highly anisotropic in copper, this dependence being shown in Fig. 2. The angular variation shows a fourfold symmetry which is distorted somewhat, probably because of unavoidable misalignment with respect

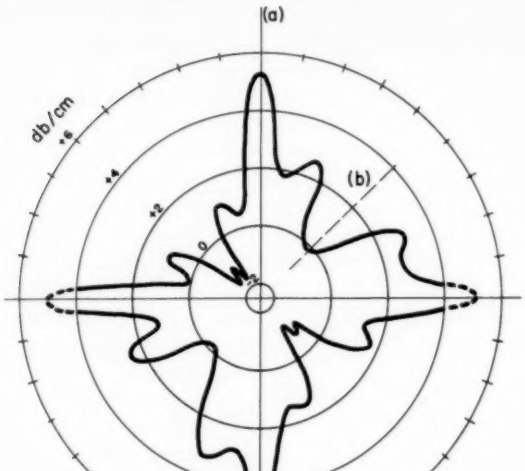

FIG. 2. Attenuation along [001] of 75-Mc/sec longitudinal wave vs the direction of H about the [001] axis, where $H = 6500$ gauss. The direction (a) corresponds to [100] and (b) to [110]; these are the directions in which the curves shown in Fig. 1 were taken. The attenuation scale is relative to that at $H = 0$.

to the magnet. The interpretation one should place on this polar pattern is not clear. Rodriguez's theory predicts in this situation that the high-field attenuation should become proportional to $\omega^2\tau$ (where ω is the angular frequency of the wave and τ is the electron relaxation time). Thus the pattern shown in Fig. 2 may reflect anisotropy of τ as well as of the Fermi surface.

Preliminary measurements also have been made of the attenuation of a plane polarized shear wave with H along the direction of propagation. As pointed out by Kjeldaas, this configuration is interesting because certain groups of electrons, selected by the value of the magnetic field, remain in resonance with the periodic fields.[5] The results at the lower frequencies agree qualitatively with Kjeldaas' predictions except that the experimental attenuation initially increases with H. The attenuation at high frequencies does not diminish with increasing H as rapidly as the theory predicts. However, this may be due to a magnetically induced rotation of the plane of polarization, an effect predicted by Kjeldaas. These results, as well as measurements in other crystallographic directions, will be discussed at more length in a subsequent paper.

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†Edgar Lewis Marston Fellow.

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SOME EFFECTS OF STRAIN IN THIN SPECIMENS ON ABSORPTION MEASUREMENTS AT LOW TEMPERATURE

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Direct transition excitons[1,2] and the magneto-optical effect[2,3] have recently been investigated in Ge by making transmission measurements on polished plates 10 microns and less in thickness, over the range from room temperature to 1.5°K under high spectral resolving power. We have already drawn attention[1] to discrepancies between our measurements and those of the Lincoln group,[3] which are much greater than the quoted limits of error. What we believe to be the explanation of these differences is now presented. It is emphasized that in order to get results which are accurately characteristic of the specimen, it must be mounted in a strain-free manner.

The discrepancies are twofold:

(a) At low temperatures the absorption line due to excitation of electrons into the ground state of the direct transition exciton is consistently observed by the Lincoln group at a higher energy than by us.

(b) The direct transition energy gaps E_0 deduced from the observations, which are in excellent agreement at room temperature,[4] diverge

progressively on cooling, the Lincoln values being the higher.

Since (b) involves differences of interpretation, we focus attention on (a) which is entirely a matter of observation.

With the willing cooperation of the Lincoln group we have been able to make measurements on one of their specimens. For convenience this was done at 77°K, and confirmed the position of the exciton line in the Lincoln group's specimen to be in good agreement with their own measurements[2] (within ~0.001 ev). We therefore discount errors on either part in spectrometer calibration and difference in specimen temperature, and turn attention to the specimens themselves. The difference between them may be intrinsic or due to their experimental environment, but as we have noticed no significant variation between specimens taken from different crystals, the latter is the more likely.

A probable explanation was in terms of strains in the specimens, which would distort the band structure and show up as a shift or broadening of the absorption edge. Since the Lincoln specimens were glued to a glass backing they would certainly be under considerable strain on cooling, owing to differential contraction, although probably comparatively free from strain at room temperature, whereas ours being unbacked and freely suspended are strain-free at all temperatures. This would account for the progressive disparity between the observations on cooling. Furthermore our measurements and the Lincoln group's are in fairly good agreement in the indirect transition region.[2,5] Here the absorption coefficient is about 100 cm^{-1} or less and thick specimens are used, which would be much more difficult to strain, even if backed.

These ideas were tested as follows:

(*i*) The Lincoln technique was duplicated as far as possible, i.e., a specimen (RRE1) glued to glass with Araldite[6] was prepared and measured at 77°K. In this, the exciton line appeared at a higher energy in rough agreement with the Lincoln value. Perfect agreement would not be expected as the strain taken up by the specimen depends on a number of details such as the expansion coefficient of the glass, the adhesive and its thickness, etc. It may be noted that using crystal quartz instead of glass produced a considerably larger shift to higher energy.

(*ii*) The position of the exciton line was observed in a freely mounted specimen (RRE2) which was then glued to a glass substrate and remeasured. The line was shifted to higher energy as expected. These results are plotted in Fig. 1. In addition to the shift it is seen that backing the specimen also distorts the form of the transmission curve. In particular an extra band (marked *a* in Fig. 1) appears. This band was noted by the Lincoln group[3] but its presence was unexplained. Our measurements indicate that it is produced by strain in the specimens and shifts along the energy scale with temperature in the same way as the absorption edge. It may well be linked with distortion of the band structure.

We have already shown[1] that the absorption coefficient of a freely mounted specimen follows Elliott's[7] theoretical predictions at energies above E_0. This is not so for the backed specimens.

(*iii*) Since the direction of shift was consistent with compressional strain,[8] a system was sought which would impose a tension on the specimen when cooled, and should therefore produce a shift in the opposite direction. Assuming the backing to be the important factor, then this should be achieved by mounting a sample on fused silica.[9] This was tried and a shift of about 0.005 ev occurred on cooling, in the direction of lower energy as expected. It is interesting that Rose-Innes[9] used a very similar system deliberately to strain samples of Ge and Si. He estimated that the stresses involved correspond to about 10^9 dyne cm^{-2}. Taking Paul and Warschauer's estimate[8] of the pressure coefficient of E_0 for Ge to be 1.2×10^{-11} ev/dyne cm^{-2} we might expect a shift of around 1.2×10^{-2} ev for this specimen configuration if all the strain is taken

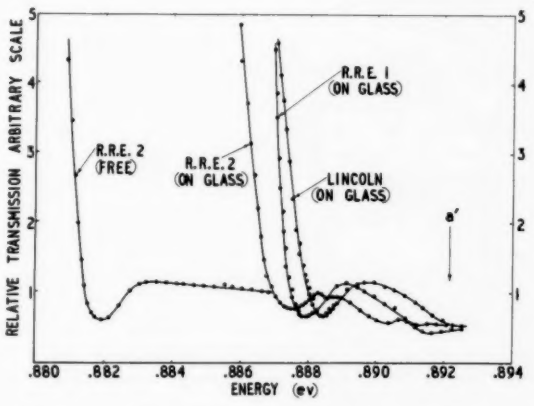

FIG. 1. Transmission ratio of various Ge specimen configurations at 77°K.

up by the specimen. This is about twice that actually observed.

The results clearly show that thin specimens must be mounted in a strain-free fashion for this type of measurement if data characteristic of the unstrained material are desired. It follows that cooling the specimen by conduction through a solid backing is unsatisfactory since it is practically impossible to arrange for the relative expansion to be zero. Recourse must be had to direct immersion in the cooling liquid, which brings its own difficulties,[10] or immersion in thermal exchange gas.[11,12] The latter method is used in this laboratory. These considerations will apply, of course, to other materials than Ge.

Strain in the specimens will also affect magneto-absorption measurements. Clearly, a uniform strain will shift the band edges and the adjacent Landau levels to roughly the same extent. The magneto-absorption bands will thus experience an appreciable shift (of the same order as that of the exciton absorption), but it is unlikely that changes in their relative positions will be detectable by the spectroscopic means currently in use. The practical situation with a backed specimen is almost certainly one of nonuniform strain and will be more complicated.

Finally it is emphasized that the condition of the specimens should be kept very much in mind when correlating data obtained from different experiments.

We are indebted to the Lincoln group for lending us two of their specimens and for communicating their results to us before publication. Our thin specimens were skillfully prepared by S. C. J. Brooke. This Letter is published by permission of H. B. M. Stationery Office.

[1]Macfarlane, McLean, Quarrington, and Roberts, Proc. Phys. Soc. (London) 71, 863 (1958).

[2]Zwerdling, Lax, Roth, and Button, Phys. Rev. (to be published). In this paper the transmission ratio of a sample at 77°K is given and can be directly compared with out results.

[3]Zwerdling, Lax, and Roth, Phys. Rev. 108, 1402 (1957).

[4]In reference 1 we state that our value for E_0 at 291°K should be regarded as less reliable than those at the lower temperatures. This is so, but should not be taken to imply that it is unreliable in the present context. The discrepancies which we are now discussing (~0.006 ev) are more than ten times as great as the error in E_0.

[5]Macfarlane, McLean, Quarrington, and Roberts, Phys. Rev. 108, 1377 (1957).

[6]A thermosetting adhesive similar to Allymer cement. We understand Allymer cement is used by the Lincoln group.

[7]R. J. Elliott, Phys. Rev. 108, 1384 (1957).

[8]W. Paul and D. M. Warschauer, J. Phys. Chem. Solids 5, 89 (1958).

[9]A. C. Rose-Innes, Proc. Phys. Soc. (London) 72, 514 (1958).

[10]V. Roberts, Colloquium Spectroscopicum Internationale VI (Pergamon Press Ltd., London, 1956), p. 508.

[11]McMahon, Hainer, London, and King, J. Opt. Soc. Am. 39, 786 (1949).

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MAGNETIC FORM FACTOR OF COBALT*

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We wish to report preliminary results on the magnetic form factor of the face-centered cubic phase of metallic cobalt, which indicate a deviation from spherical symmetry for its $3d$ electrons. These measurements represent a continuation of studies on the angular dependence of the magnetic scattering in the transition elements using a polarized neutron beam spectrometer[1]; the results on iron and nickel have been already reported in reference 1. These studies were undertaken because of the current interest in the $3d$ electron configurations in the transition elements and the pertinence of magnetic form factor data to such discussions.

The sensitivity of the polarized beam technique in establishing the absolute magnitude of the magnetic scattering depends on the ratio of the nuclear scattering amplitude to the magnetic scattering amplitude at the different scattering angles. For cobalt, this ratio is very favorable and it is possible to obtain sizable differences in the reflectivities for beams of different polarization even for large scattering angles, where the magnetic scattering is greatly reduced. For this reason every effort was made to reduce the in-

trinsic experimental uncertainties present in this type of measurement to a minimum so as to ascertain whether in cobalt the angular dependence of the magnetic scattering gives any definite indication of structure in the distribution of unpaired electrons.

The method used to obtain the absolute magnetic scattering amplitude has been described in detail in reference 1. Briefly, the ratio of reflected intensities from a magnetized single crystal is measured for incoming neutrons polarized first parallel to the field on the crystal, and then antiparallel. This polarization ratio for a particular Bragg reflection depends on the ratio p_{hkl}/b_{hkl}, where p_{hkl} and b_{hkl} are the magnetic and nuclear structure factors for the (hkl) reflection. Since b_{hkl} is known, the measurement yields p_{hkl}. It is necessary to correct the observed values of the polarization ratio for the presence of extinction in the test crystal and for the depolarization of the neutron beam as it traverses the crystal. The latter was measured by a method given in reference 1, and was shown to be less than 1%. Extinction, although small, was estimated by performing the measurements for crystals of different thicknesses and at two different neutron wavelengths (0.89A and 1.04A).

The test crystal for the fcc measurements consisted of a pillar crystal of the alloy $Co_{0.92}Fe_{0.08}$, the presence of the iron serving to stabilize the cobalt in the fcc phase. All measurements were made at room temperature. The results of the measurements taken to date are shown in Table I and plotted in Fig. 1. In calculating the magnetic scattering amplitudes we have used the values $b_{Co} = 0.25 \times 10^{-12}$ cm ^{2} and $b_{Fe} = 0.96 \times 10^{-12}$ cm.

Table I. Magnetic form factor of face-centered cubic cobalt.

Reflection	$\sin\theta/\lambda$	p_{hkl}/b_{hkl}	f
(111)	0.244	1.21 ±0.06	0.781±0.040
(200)	0.283	1.01 ±0.06	0.652±0.040
(220)	0.397	0.575±0.005	0.371±0.003
(311)	0.467	0.418±0.003	0.270±0.002
(222)	0.487	0.350±0.002	0.266±0.002
(400)	0.563	0.286±0.006	0.184±0.004
(331)	0.613	0.166±0.002	0.107±0.002
(422)	0.689	0.091±0.006	0.059±0.004
(333)	0.731	0.054±0.006	0.035±0.004
(440)	0.796	0.034±0.003	0.022±0.002

Of particular interest in the data is the failure of the form factor values for some of the higher (hkl) reflections to fall on a smooth curve, in particular the (311) and (400) reflections. In order to emphasize that these "bumps" are real, we have plotted in Fig. 2 the outer portion of the form factor curve on an expanded scale along with the estimated uncertainties in the measurements. These "bumps" have led us to suggest a deviation from spherical symmetry for the $3d$ electrons. As far as we know, these measurements give the first experimental indication of such a departure. A somewhat similar behavior to the above had also been noticed in the fcc nickel form factor. However, because of the low magnetic scattering amplitude of nickel, espe-

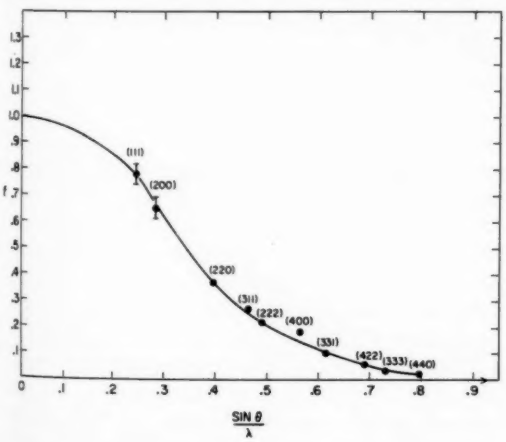

FIG. 1. Magnetic form factor of face-centered cubic cobalt.

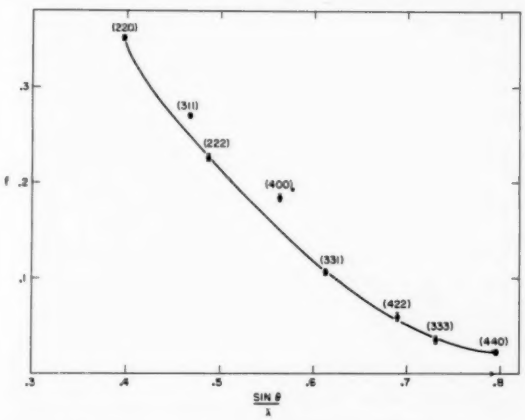

FIG. 2. Outer portion of the magnetic form factor curve of fcc Co.

cially at the higher scattering angles, it was difficult to place these observed "bumps" on a more definite basis.

Because of the change in the symmetry, it would also be of interest to compare the magnetic form factor of hexagonal close-packed Co with that of face-centered cubic Co. Hence we are continuing these measurements in order to refine the fcc data further and to determine the form factor of the hexagonal phase. A detailed report will be submitted upon the completion of this work.

We are indebted to Dr. T. A. Kaplan and Dr. S. J. Pickart for profitable discussions of these results. We also wish to thank Dr. H. J. Williams and Dr. R. M. Bozorth of Bell Telephone Laboratories for supplying us with the crystals.

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*This work was performed under the auspices of the U. S. Atomic Energy Commission and the National Security Agency.

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[2]W. L. Roth, Phys. Rev. 110, 1333 (1958).

LATTICE VIBRATIONS IN SILICON AND GERMANIUM

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(Received February 13, 1959; revised manuscript received March 2, 1959)

The dispersion curves of lattice waves travelling in the [001] directions of a silicon single crystal have been measured using neutron scattering. The frequency (ν) and wave vector ($\vec{q}$) of each phonon, and the character of each branch, were determined as previously discussed.[1] The complete results for the longitudinal acoustical (LA), longitudinal optical (LO), and the degenerate transverse acoustical (TA) and transverse optical (TO) branches are shown in Fig. 1.

Two series of experiments were performed using a new multiple axis crystal spectrometer at the NRU reactor. In the first, neutrons of 1.900A wavelength were scattered from a single crystal of silicon at angles in the vicinity of 90° with the crystal oriented so that the wave vector of the outgoing neutrons ($\vec{k}'$) lay very nearly along the line $(\bar{2}, \bar{2}, 3)$ to $(\bar{2}, \bar{2}, 6)$ of the silicon reciprocal lattice.[2] The wave vectors of all phonons observed in these experiments thus lay in the [001] direction of the reduced zone. In the second series, neutrons of 1.900, 1.350, and 1.693 angstroms were used in the method of successive approximations,[1] in which phonons were measured in regions of reciprocal space in which their polarization, and optical or acoustical character, could be unambiguously determined. All measurements were made at room temperature. The points are believed to define the level of all branches to about 3%. Values of important frequencies (units 10^{12} cps) are: Raman ($q=0$) 15.3 ± 0.3, TO[001] 14.2 ± 0.3, L[001] 11.9 ± 0.5, TA[001] 4.35 ± 0.15.

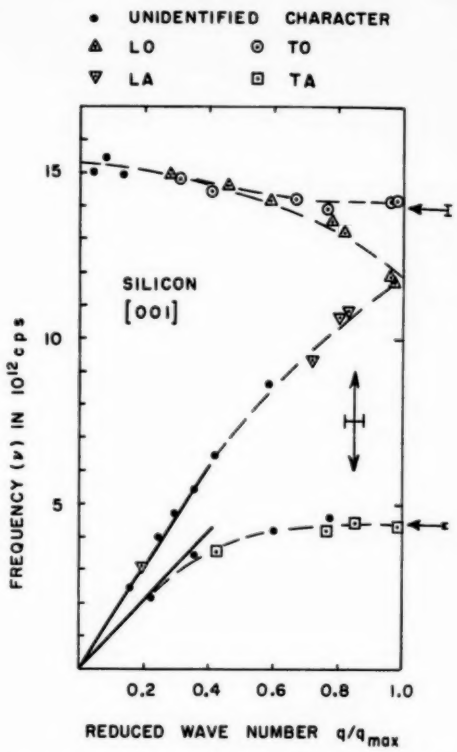

FIG. 1. Frequency vs reduced wave number for lattice vibrations propagating in the [001] direction of the reduced zone in silicon at room temperature. The heavy lines have the slope of the appropriate velocity of sound as found by ultrasonic measurements [H. J. McSkimmin, J. Appl. Phys. 24, 988 (1953)]. The vertical arrow indicates the presumed position (q_0) of the minimum of the conduction band (reference 13). The horizontal arrows indicate the frequencies of phonons at q_0 found from the indirect transition (references 10-12).

The measurements are in disagreement with x-ray measurements of Learn.[3] The Raman frequency is in agreement with a prediction[4] 15.45 $\times 10^{12}$ cps based on the far infrared spectrum, and with an independent neutron measurement of 15.2×10^{12} cps by Palevsky, Hughes, Kley and Tunkelo.[5] The dispersion curves are in agreement with predictions[4,6] from specific heat, elastic constant, and far infrared measurements, that the frequencies for silicon are similar to those for germanium at similar positions in the zone but higher by a factor of about 1.75. Six figures of merit obtained from these data gave[4] an average ratio 1.745 with a standard deviation of 0.04. The average ratio of the silicon frequencies reported herein to the frequencies of germanium measured by Brockhouse and Iyengar[1] at the same positions in the reduced zone is 1.73 ± 0.04. If exact homology held between silicon and germanium, the harmonic force constants would be proportional to the inverse square of the lattice constant (a), and the frequencies therefore proportional to $(Ma^2)^{-1/2}$, where M is the mass of the atom. Thus the frequency ratio would be 1.675, slightly but significantly lower than the measured ratios. The expected frequency ratio can also be computed from the melting temperatures using the Lindemann law,[7] leading to a value of 1.96. The lack of complete homology thus seems to be connected with an abnormally low melting point in germanium (compared with silicon).

It follows from the approximate homology that in silicon, as in germanium, either long-range forces between atoms are important or the Born-Von Kármán theory is inadequate.[1,8,9]

The [001] direction was selected for initial study because the minimum of the conduction band lies in this direction, and it was hoped that the wave vector ($\tilde{q}_0$) of the minimum could be located by comparison of these results with other types of measurements which are dependent on the value of $\tilde{q}_0$. Measurements of the infrared absorption spectrum by Macfarlane, McLean, Quarrington, and Roberts[10] have been interpreted to give values for the frequencies at q_0 of 4.4, 14.0, 21.9, and 29.5×10^{12} cps. Measurements of the spectrum of recombination radiation by Haynes, Lax, and Flood[11] originally gave frequencies a little lower than those obtained by Macfarlane et al., but more recent measurements are in agreement.[12] The two higher frequencies of each set are clearly inconsistent with the results of Fig. 1 but could arise from double phonon processes as suggested[4] earlier. From measurements of electron nuclear double resonance, Feher[13] has found that $q_0/q_{max} = 0.85 \pm 0.03$. If this value is accepted, then the two lower frequencies of each set must be TA and probably TO. The LO and LA transitions are presumably weak and overshadowed by the very strong TO transition.

The measurements of lattice frequencies of germanium of Brockhouse and Iyengar,[1] discussed above, and those of Ghose, Palevsky, Hughes, Pelah, and Eisenhauer[14] are in disagreement for the TA and especially for the longitudinal branches near the zone boundary in the [001] direction. For L[001] Brockhouse and Iyengar quoted $(6.9 \pm 0.4) \times 10^{12}$ cps, while the corresponding frequency derived from the measurements of Ghose et al. is $(5.3 \pm 0.4) \times 10^{12}$ cps. In an effort to resolve this question we have remeasured[15] the [001] longitudinal branch in germanium using neutrons of longer wavelength (1.917A) than those used by Brockhouse and Iyengar (1.52A), and somewhat higher resolution. The two types of experiments discussed above for silicon were performed, and gave results in agreement with one another. The dispersion curve (nine points) obtained agreed with that of Brockhouse and Iyengar, the frequency of the L[001] phonon being $(7.0 \pm 0.3) \times 10^{12}$ cps.

The author wishes to thank Dr. J. R. Haynes, Dr. M. Lax, Dr. J. Burton, and Dr. M. Tanenbaum of Bell Telephone Laboratories for the loan of the crystals.

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[1] B. N. Brockhouse and P. K. Iyengar, Phys. Rev. 111, 747 (1958).

[2] See Fig. 4 of reference 1.

[3] A. Learn, thesis, Massachusetts Institute of Technology, 1958 (unpublished). See also H. Cole and E. Kineke, Phys. Rev. Lett. 1, 360 (1958).

[4] B. N. Brockhouse, paper given at the International Conference on Semiconductors, Rochester, New York, August 18-22, 1958 [J. Phys. Chem. Solids (to be published)].

[5] Palevsky, Hughes, Kley, and Tunkelo, this issue [Phys. Rev. Lett. 2, 258 (1959)].

[6] Similar homology arguments have been presented in the past; see references quoted in reference 4 and F. Herman (private communication).

[7] See N. F. Mott and H. Jones, Properties of Metals and Alloys (Oxford University Press, London, 1936), p. 13.

[8] F. Herman, paper given at Conference on Semiconductors (see reference 4).

[9] M. Lax, Phys. Rev. Lett. 1, 133 (1958). The arguments of Lax on silicon are still applicable even though

the actual phonon frequencies are greatly different from those he used, which were those given by the infrared workers.

[10]Macfarlane, McLean, Quarrington, and Roberts, Phys. Rev. 111, 1245 (1958); paper given at Conference on Semiconductors (see reference 4).

[11]Haynes, Lax and Flood, Bull. Amer. Phys. Soc. Ser. II, 3, 30 (1958); paper given at Conference on Semiconductors (see reference 4).

[12]J. R. Haynes (private communication).

[13]G. Feher, paper given at Conference on Semiconductors (see reference 4).

[14]Ghose, Palevsky, Hughes, Pelah, and Eisenhauer, Phys. Rev. 113, 49 (1959).

[15]These experiments were performed after the initial submittal of this Letter.

LATTICE VIBRATIONS IN SILICON BY SCATTERING OF COLD NEUTRONS*

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(Received February 18, 1959)

The use of subthermal or "cold" neutrons for investigating the lattice vibrations in solids has proved to be of increasing value in the past several years. Upon absorption of a phonon (the quantum of lattice vibration energy) while scattering, the neutron energy changes by such a large amount that the phonon energy can usually be accurately fixed. The method is particularly useful for the optical vibrations, as shown by the first measurements[1] of high-frequency modes in zirconium hydride and germanium, cases for which infrared work had given inconclusive results. In the light of similar uncertainty concerning the optical (Raman) frequency in silicon, we have determined it, as well as one acoustical branch, by use of cold-neutron scattering.

Because of the great interest in the electronic properties of silicon, a number of infrared measurements have been performed, which have been used to infer the Raman frequency, the optical vibration of zero phonon momentum. For example, Collins and Fan,[2] on the basis of infrared absorption measurements, conclude that $h\nu$ for the Raman line is 0.076 ev. Haynes, Lax, and Flood,[3] from a measurement of recombination lines from infrared excitation, obtain optical lines between 0.08 and 0.12 ev. Macfarlane, McLean, Quarrington, and Roberts,[4] on the basis of infrared absorption edge measurements, report similar values, 0.09 and 0.12 ev. Recently, Cole and Kineke[5] have reported that the unpublished x-ray results of A. Learn give a value of 0.071 ev. On the theoretical side, Hsieh,[6] using the theory of Smith,[7] has calculated the optical vibration spectrum from the measured force constants and obtained a Raman value of 0.077 ev, using only first neighbor interactions, and a value of 0.10 ev when the effects of second neighbors are included. These discordant results indicate the need for an unambiguous value of the optical line, such as that obtainable from cold-neutron scattering.

In the cold-neutron technique as developed at Brookhaven,[8] the neutron beam incident on the scatterer is produced by filtration through 8 inches of beryllium. No Bragg (elastic) scattering can occur for the low-energy (~0.005 ev) incident neutrons, and those scattered at 90° (the position of the detector) are of necessity those that have gained energy by interacting with the lattice vibrations. Since the energy of the optical vibrations of silicon is in the neighborhood of 0.05 ev, the neutrons experience a tenfold increase in energy when scattered (Fig. 1). It is this large change in energy, giving directly the phonon energy, that is a distinct advantage of the cold-neutron method.

In the present measurements, the incident neutron beam was perpendicular to the (100) plane of the silicon single crystal. The energy of neutrons scattered at 90° was determined by measuring their time of flight over a 5-meter flight path, utilizing a slow chopper. Figure 1 shows a typical set of experimental data, plotted in terms of time of flight. The sharp peak at high energy represents the absorption of one quantum of vibrational energy from an optical mode, and the one at lower energy corresponds to an acoustical mode. Data similar to Fig. 1 were obtained for a number of orientations of the crystal as it was rotated about the [100] axis. The results give energies for phonons of various momenta $\vec{q}$, with $\vec{q}$ limited to the (100) plane of reciprocal lattice space. Points for $\vec{q}$ in the [110] direction, extrapolated to $\vec{q}=0$, give the energy of the Raman line as 0.063 ± 0.003 ev (phonon frequency, ν, of 15.2×10^{12} cps); and at the edge of the Brillouin zone (maximum $\vec{q}$) in the [110] direction the transverse acoustic branch has an energy of 0.020 ± 0.002 ev ($\nu = 4.84\times10^{12}$ cps). Thus our value for the Raman line is not in agreement with the infrared measurements nor with the Hsieh calculations. It is possible, however, to reinter-

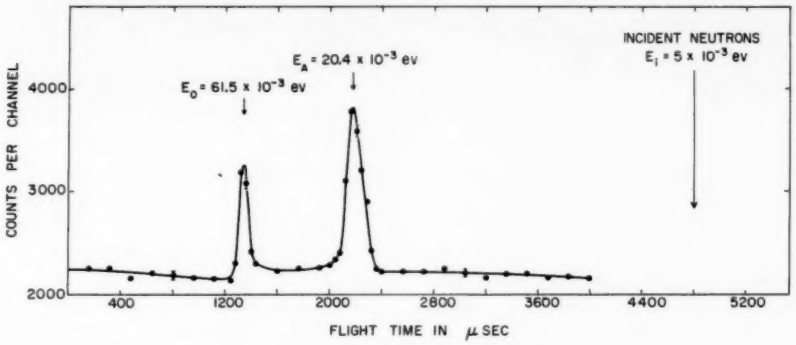

FIG. 1. Counting rate as a function of time of flight for neutrons scattered at 90° by Si. The arrow indicates the position of the incident neutrons on the same scale. The peaks marked E_0 and E_A are the energies of scattered neutrons which have gained one quantum of vibrational energy from the optical and acoustical modes, respectively.

pret the high-energy infrared results in terms of combination lines so that they would be consistent with the neutron measurements.

Our purpose in measuring one acoustical branch, in addition to the Raman line, was to check the validity of a simple model suggested about two years ago by Herman,[9] who argued that in the first approximation the vibration frequencies of Si and Ge should be inversely proportional to the square root of the masses of the vibrating atoms, for they both have the same valence electron structure. This ratio is 1.61, and a comparison of the present results with our Ge measurements[10] gives a ratio of 1.69 ± 0.10 for the Raman frequencies ($\vec{q} = 0$) and a ratio of 1.61 ± 0.15 for the transverse acoustic phonons at the edge of the Brillouin zone in the [110] direction. Thus an extremely simple physical model serves to correlate the frequencies of Si and Ge within the present experimental error, for both optical and acoustical modes. The failure of Hsieh's calculation of the vibration frequencies, and the proportionality of Si to Ge frequencies, indicate that for Si, as well as Ge,[11] there are long-range forces present.

Note added in proof.—The results for the Raman frequency in this Letter and those of Brockhouse in the accompanying Letter (using neutrons of higher energy) are in excellent agreement, for both silicon and germanium. There are unexplained differences for the acoustic branches, for which some of our frequencies are higher, and some lower than his. The differences, however, do not affect the conclusion that the ordinary Born–Von Kármán theory using short-range forces only is inadequate, which follows from either set of results.

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*Work performed under the auspices of U. S. Atomic Energy Commission.

†Guest scientist from University of Freiburg, Freiburg, Germany.

‡Guest scientist from Finland Institute of Technology, Helsinki, Finland.

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CONTRIBUTION OF THE NUCLEUS TO THE SPECIFIC HEAT OF RHENIUM*

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(Received February 25, 1959)

Measurements of the heat capacity of a Re ingot[1] between 0.37 and 4.2°K show an anomalous contribution to the specific heat which is proportional to T^{-2}. This temperature dependence suggests that the contribution is the high-temperature part of a Schottky-type anomaly, arising from excitations among a set of discrete energy levels. Figure 1 shows the ratio of the total specific heat to the absolute temperature, C/T, plotted as a function of T. The specific heat of the Re measured in zero magnetic field can be expressed as

$$C_n = \gamma T + \alpha T^3 + A_0 T^{-2}, \quad T > 1.85^\circ\mathrm{K} \tag{1}$$

$$C_s = C_{es} + \alpha T^3 + A_0 T^{-2}, \quad T < 1^\circ\mathrm{K} \tag{2}$$

where γT is the electronic specific heat in the normal state, αT^3 is the specific heat of the lattice, and $A_0 T^{-2}$ is the anomalous contribution. C_{es} is the electronic specific heat which is expected, according to the Bardeen, Cooper, and Schrieffer theory of superconductivity,[2] to be given by

$$C_{es} = \gamma T_c a \exp(-bT_c/T), \quad T_c/T > 2. \tag{3}$$

Because αT^3 is small throughout and $A_0 T^{-2}$ is small at the higher temperatures, the constants can readily be adjusted by successive approximations to fit the data. Accepting the validity of Eq. (3), we find $A_0 = 0.052 \pm 0.002$ millijoule deg/mole. The other constants are $\gamma = 2.31 \pm 0.02$ millijoule/mole deg^2 and $\alpha = 0.026 \pm 0.002$ millijoule/mole deg^4, from which it follows that the Debye temperature $\theta_0 = 417 \pm 10$°K. With this choice of constants, the data appear to be consistent with the second law of thermodynamics, i.e., with

$$\left(\int_0^{T_c} C_{es} T^{-1} dT = \int_0^{T_c} \gamma dT \right)$$

within the limits of experimental accuracy. Setting $T_c = 1.70$°K in Eq. (3), we obtain $a = 8.5$ and $b = 1.46$.

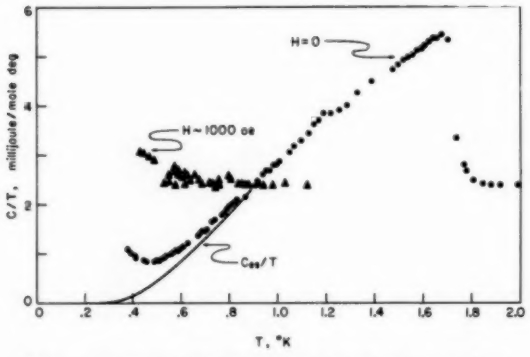

FIG. 1. Ratio of the specific heat of rhenium to absolute temperature as a function of temperature in zero magnetic field (●) and in 1000 oe (▲). The solid line is C_{es}/T vs T.

To quench the superconductivity and observe the magnetic field dependence of the anomaly, we remeasured in a field of about 1000 oersted. The resulting specific heat is also plotted in Fig. 1, and can be expressed as $\gamma T + \alpha T^3 + A_H T^{-2}$ with $A_H = 0.06 \pm 0.01$ millijoule deg/mole (because of fluctuations in the applied field, this measurement was less accurate). A_H does not appear significantly different from A_0, and we conclude that the anomaly is influenced by the applied field less than should be expected if it were caused by repopulation of electronic energy levels. For, since no deviation from the T^{-2} dependence of the anomaly is observed down to 0.37°K, the characteristic temperature of the zero-field splitting is certainly less than 0.37 deg. Calculations for a variety of splitting schemes, which involve interactions of the relatively large electronic magnetic moment with the field H, yield for $H = 1000$ oersted an anomaly about twice the magnitude of the one we observe in zero field.

There remains the possibility that the energy levels of the nucleus are separated sufficiently to produce the anomaly. This splitting can be produced by interaction between the nuclear electric quadrupole moment and the crystalline electric field.[3] Because the nuclear magnetic moment is much smaller than the electronic moment, its interaction with the magnetic field is much smaller than in the electronic case, and leads to a relatively weak dependence of the specific heat on H. The nuclei of the two stable Re isotopes have a spin of 5/2 and an average quadrupole moment $Q = 2.7 \times 10^{-24}$ cm^2. In the presence of an axially symmetric field gradient, whose maximum value we denote by eq, six degenerate energy levels of the Re nucleus will be split into three doubly degenerate pairs,[4] the

separation of the lower two being $3e^2qQ/20$ and of the upper two, $6e^2qQ/20$. This splitting scheme leads to a specific heat $A_0T^{-2}=(14/9)Ra_0^2T^{-2}$ in the high-temperature limit $T>>a_0$, where a_0 has been written for $3e^2qQ/20$. Comparing this result with the experimental value of A_0, we obtain $a_0=0.002$ deg and the maximum field gradient $eq=0.14\times10^{16}$ esu. Since the ingot is polycrystalline and contains impurities, the value of eq may differ from that of a perfect crystal.

*Supported by a Signal Corps Contract.

[1]This ingot, furnished to us by Dr. M. P. Garfunkel, has a history similar to specimen 4c of J. K. Hulm and B. B. Goodman [Phys. Rev. 106, 659 (1957)], for which: T_c = 1.81°K with a transition breadth of 0.05 deg. The known impurities are Fe = 0.0011%, Si < 0.01%, and $W\sim0.03\%$. We are grateful for the loan of this sample.

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[3]See, for instance, M. H. Cohen and F. Reif, Solid State Physics, edited by F. Seitz and D. Turnbull (Academic Press, New York, 1957), Vol. 5, p. 321.

[4]T. P. Das and E. L. Hahn, Solid State Physics, edited by F. Seitz and D. Turnbull (Academic Press, New York, 1957), Supplement 1.

NUCLEAR SPECIFIC HEAT OF GALLIUM AND ZINC

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(Received February 25, 1959)

Recently the specific heats of gallium[1] and zinc[1-3] have been measured in the superconducting state below 1°K. However the data have all been analyzed without concern of the possible effect of an interaction of the nuclear quadrupole moment with the gradient of the crystalline field. Now that such an effect has been shown to be of considerable importance for the specific heat of rhenium,[4] it becomes necessary to review the previous results.

Knight et al.[5] measured the pure quadrupole resonance spectra for Ga^{69} and Ga^{71} in the metallic state. From their results it becomes possible to calculate the quadrupole coupling constant for both isotopes and also the contribution of the nuclei to the specific heat of naturally occurring gallium (60.2% Ga^{69}, 39.8% Ga^{71}). This contribution is given by $C=4.6/T^2$ ergs/mole deg when $T>>0.001$°K. It is only 5% of the specific heat of the conduction electrons at the lowest temperatures measured by us. Therefore we did not observe a departure of the electronic specific heat in the superconducting state from the expected exponential decrease with temperature:

$$C_{es}=\gamma T_c a\exp(-bT_c/T). \quad (1)$$

Recalculating our data, after subtracting this nuclear term, the constants for Ga in Eq. (1) change: a from 7.0 to 7.5 and b from 1.35 to 1.38. The fit of the experimental data and Eq. (1) is extended to somewhat higher temperatures: $T_c/T>1.6$.

Zinc has a hexagonal crystal, and hence an appreciable field gradient can exist at the nuclei. However the single isotope which is magnetic, Zn^{67} (spin 5/2), has a natural abundance of only 4%. Phillips[2] noted deviations of the specific heat from the exponential temperature dependence, Eq. (1), outside the experimental error. It is reasonable to assume that these deviations are nuclear in origin and that they have a temperature dependence proportional to $1/T^2$. From his graph it follows that the additional term is then given by $C\sim1.5/T^2$ erg/mole deg for naturally occurring zinc. Hence for Zn^{67}, $C\sim37/T^2$ erg/mole deg and the nuclear quadrupole coupling constant for Zn^{67} is approximately 70 Mc/sec. If the additional term is subtracted from the specific heat data of Phillips, his constants of Eq. (1) change: a from 5.8 to 6.2 and b from 1.22 to 1.24 (using his values of γ and T_c). We reported values $a=6.4$, and $b=1.27$, which are only slightly higher. They were obtained at higher temperatures where this additional term can be neglected.

In the normal state, at the lowest temperatures Phillips notes a 5% deviation from the expected value of the heat capacity but allows the possibility of errors in his thermometers. However, this value of the deviation is to be expected if the splitting of nuclear energy levels of the Zn^{67} isotope contributes to the specific heat. It should be possible to substantiate our assumption by a quadrupole resonance experiment.

It is difficult to obtain data from Zavaritskii's[3] curves for zinc, but it appears that his results are not inconsistent with the above conclusions.

Zavaritskii[3] uses the relation $a=4b^2$ for the

constants in Eq. (1). This relation holds very well for all the elements[1] for which these constants are known.

We would like to use this opportunity to correct a statement in our previous article.[1] Dr. D. E. Mapother pointed out that on cooling through the transition temperature in a magnetic field a superconducting shell does not expel flux lines. Therefore our measurements were performed in the earth's magnetic field.

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*National Science Foundation Postdoctoral Fellow.

[1]G. Seidel and P. H. Keesom, Phys. Rev. 112, 1083 (1958).

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[3]N. V. Zavaritskii, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 1116 (1958)[translation: Soviet Phys. JETP 34, 773 (1958)].

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THREE-LEVEL MASERS AS HEAT ENGINES*

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(Received January 16, 1959)

The purpose of this note is to demonstrate that three-level masers[1,2] can be regarded as heat engines. The principal conceptual difference between these and conventional heat engines is that in the 3-level maser one is concerned with the discrete energy levels of a particle's internal energy whereas in a conventional heat engine one is concerned with the continuous spectrum of energies associated with external motion of the working substance. In treating a 3-level maser as a prototype of heat engine, a particular advantage is, in our opinion, the resulting conceptual simplicity. Especially, it is easily shown that the limiting efficiency of a 3-level maser is that of a Carnot engine.

Consider the system shown in Fig. 1. A three-level system is assumed with all transitions allowed and with no appreciable relaxation processes. The usual 3-level maser terminology is introduced by correlating transition 1 — 3 with pump frequency ν_p, 1 — 2 with signal frequency ν_s, and 2 — 3 with idler frequency ν_i. As a further convention, the length of each energy level line is drawn proportional to its population.

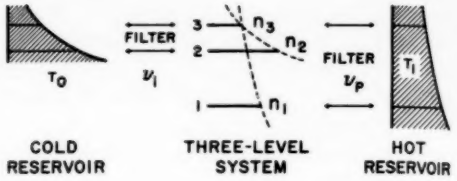

FIG. 1. Three-level system in thermal contact with two heat reservoirs.

The levels 1 and 3 are in thermal contact, through a filter passing frequencies in the vicinity of ν_p and rejecting frequencies in the vicinity of ν_i and ν_s, with a heat reservoir at temperature T_1. The temperature is indicated in the figure by showing schematically the Boltzmann distribution of this heat reservoir. Levels 2 and 3 are in thermal contact with a reservoir at a lower temperature T_0 through a filter which passes frequencies in the vicinity of ν_i but rejects those close to ν_p and ν_s.

Experimentally, the high-temperature reservoir might be realized by a gas noise lamp and the filter by a wave guide cutting off the lower frequencies. For practical purposes, however, the single mode present in a wave guide does not provide good thermal contact. The assumed coupling situation to the low-temperature reservoir, on the other hand, was closely approximated by experimental conditions in some maser experiments.[3] There, the idler transition of the gadolinium three-level system was coupled, through spin-spin interaction at frequency ν_i, to a transition of the same frequency of cerium ions within the same crystal. Thus, through the resultant short spin-lattice relaxation time, good thermal contact to the lattice heat reservoir at T_0 was established.

In the system described, for each quantum $h\nu_p$ supplied by the hot reservoir, the energy $h\nu_i$ is passed to the cold reservoir. The smaller quantum $h\nu_s$ can be extracted at the signal transition if maser action prevails, that is if $n_2/n_1 \geq 1$. Thus the efficiency of this idealized system in maser operation is

$$\eta_M = \nu_s/\nu_p. \tag{1}$$

From the Boltzmann factors involved, we find

$$\frac{n_2}{n_1} = \frac{n_2}{n_3}\frac{n_3}{n_1} = \exp\left(\frac{h\nu_i}{kT_0}\right) \times \exp\left(-\frac{h\nu_p}{kT_1}\right). \tag{2}$$

After rearrangement, this becomes

$$\frac{n_2}{n_1} = \exp\left[\frac{h\nu_S}{kT_0}\left(\frac{\nu_p}{\nu_S}\,\frac{T_1 - T_0}{T_1} - 1\right)\right]. \quad (3)$$

In this formula, one recognizes the maser efficiency η_M and the efficiency of the Carnot cycle,

$$\eta_C = (T_1 - T_0)/T_1. \quad (4)$$

Using these, the condition for maser action is

$$\eta_M \leq \eta_C. \quad (5)$$

This may be regarded as another formulation of the second law of thermodynamics. Maser efficiency equals that of a Carnot engine if the signal transition is at the verge of inversion, $n_2 - n_1 \to +0$ or $T_{sig} \to -\infty$.

As any heat engine, this system should be reversible so that it acts as a refrigerator. This is indeed the case. Suppose a quantum $h\nu_S$ is applied to the signal transition. It causes an ion to go from state 1 to 2. The ion may further jump to state 3 if the energy $h\nu_i$ is supplied by the cold reservoir. The cycle is finally completed when the ion returns from state 3 to state 1 while the energy $h\nu_p$ is communicated to the hot reservoir. In this process, energy is extracted from the idler transition, that is from the cold reservoir, so that it is refrigerated. The scheme outlined here, however, requires the signal transition to be absorptive. Otherwise the first step, application of $h\nu_{sig}$ to the signal transition, would not have been possible. Thus the refrigeration scheme is possible if $n_2/n_1 \leq 1$. Again, the limiting efficiency of the refrigerator is that of a Carnot engine and it is realized with $n_2 - n_1 \to -0$ or $T_{sig} \to +\infty$.

It seems probable at this time that generation of microwaves through thermal excitation by two temperatures will be possible experimentally. Such a scheme should be very attractive for high microwave signal frequencies. Refrigeration experiments, on the other hand, as applied to interacting nuclear and electronic spin systems have been suggested by the theoretical work of Overhauser.[4] A thermodynamical analysis of the Overhauser effect has been given by Brovetto and Cini[5] and Barker and Mencher.[6]

Finally, we should like to point out that the possibility of treating masers as heat engines sets a fundamental distinction between these and parametric amplifiers. Three-level masers are capable of operating with noise-like excitation in all three transitions. Parametric amplification requires some phase coherence, that is, monochromatic excitation is necessary for at least one of the three frequencies involved. This statement excludes heat as the source of energy for parametric amplification.

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*This work is partially supported by the Signal Corps.

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[6]W. A. Barker and A. Mencher, Phys. Rev. 102, 1023 (1956).

GAMMA-RAY ACTIVATION OF CARBON*

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(Received February 26, 1959)

A direct measurement of photonuclear reaction cross sections can be accomplished by the use of the monochromatic capture gamma rays of the $T(p,\gamma)He^4$ reaction.

If the gamma-ray energy be varied by changing the energy of the captured particle, the photonuclear cross section can be determined as a function of photon energy. The capture of protons in tritium produces a high-energy gamma ray of $(19.82 + \frac{3}{4}E_p)$-Mev energy, where E_p is the proton energy. This gamma ray has a smooth yield curve with no resonances, but it is of rather low intensity. Nevertheless, by using a highly effective detector we have measured the direct photoactivation of carbon by the $C^{12}(\gamma,n)C^{11}$ reaction in the photon energy range 20.2 to 21 Mev.

The tritium was absorbed in a thin layer of zirconium evaporated on a silver backing[1] which was water cooled. The thickness of the target for 1-Mev protons was measured to be 70 kev by observing the $T(p,n)$ threshold and comparing with very-thin-target differential curves.[2]

The carbon was reactor-grade graphite in the form of an annular ring surrounding the tritium target. The carbon subtended the angles from 90° to 102° with respect to the proton direction which corresponds to a range of photon energies

of about 50 kev due to the Doppler effect. These effects produced a spread in gamma-ray energies, as shown in the upper part of Fig. 1, of about 81 kev at 20.2 Mev, 67 kev at 20.5 Mev, and 76 kev at 21 Mev.

After a 40-minute irradiation at a given photon energy, the carbon disk was inserted in a positron detector and counted for 20 minutes. Each point in Fig. 1 was the result of about 10 such runs. The positron detector consisted of two sodium iodide crystals 5 in. in diameter by 2 in. thick placed $\frac{1}{2}$ in. apart, each viewed by a 6364 Dumont photomultiplier tube, the pulses amplified, channeled between 380 and 600 kev, and put into coincidence. This arrangement, with an 18-in. iron and lead shield around it, had a background of 3 counts per minute and an efficiency for positrons of 10% at the center tapering off smoothly to zero at the edge.

The 20-Mev gamma rays were monitored by a large sodium iodide crystal 3 in. in diameter by 4 in. thick. Pulses greater than 16 Mev and less than 22 Mev were counted. The amplifier and discriminator were calibrated on every run by recording the pulse-height distribution.

The results of our measurements are given on the graph of Fig. 1. It is noticeable that no sharp resonances of the sort reported from betatron photoneutron yield curves[3] are evident in the energy region examined. This is consistent with the total absorption measurements made in this laboratory on carbon with these monochromatic gamma rays.[4] It is also consistent with self-absorption measurements[5] in carbon.

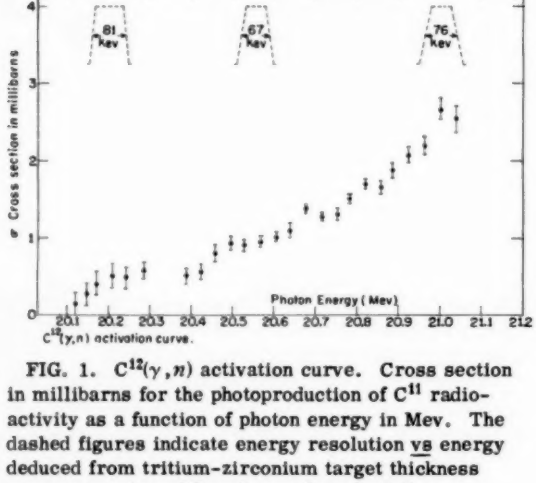
81 kev
67 kev
76 kev
σ Cross section in millibarns
Photon Energy (Mev)
20.1 20.2 20.3 20.4 20.5 20.6 20.7 20.8 20.9 21.0 21.1 21.2
C12(γ,n) activation curve.

FIG. 1. $C^{12}(\gamma,n)$ activation curve. Cross section in millibarns for the photoproduction of C^{11} radioactivity as a function of photon energy in Mev. The dashed figures indicate energy resolution vs energy deduced from tritium-zirconium target thickness measurements and Doppler angle calculations.

We would conclude from these measurements that the levels excited by photon absorption at this excitation energy (presumably mostly 1^- levels) have a level width approximately equal to their spacing or a spacing comparable with our resolution, or they could have an integrated cross section less than 0.05 Mev-mb.

The help of Professor John Winhold in the initial stages of planning this experiment is gratefully acknowledged.

*Supported by the joint program of the U. S. Atomic Energy Commission and the Office of Naval Research and by the National Science Foundation.

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[2]Jarvis, Hemmendinger, Argo, and Taschek, Phys. Rev. 79, 932 (1950).

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NUCLEON-NUCLEON SPIN-ORBIT INTERACTION POTENTIAL*

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(Received February 2, 1959)

A recent publication[1] reports the effect of a change in the spin-orbit interaction used by Signell and Marshak[2] and compares the results with the phenomenologic fits of Stapp, Ypsilantis and Metropolis.[3] Calculations with the shortened range of the spin-orbit potential have been available at Yale for some time but have not been published in view of questionable agreement with experiment. Additional calculation confirms the view that the representation of data by the potential is in many respects not altogether satisfactory and indicates that the new fits to 310-Mev data[1] do not appear reasonable in the light of the phase-shift energy dependence between 150 and 310 Mev and of disagreement with experiment within several times the experimental error in several cases between 142 and 345 Mev.

In Fig. 1 are shown phase shifts for the shortened range of spin-orbit interaction denoted as MS 1 and for the original Marshak-Signell case,

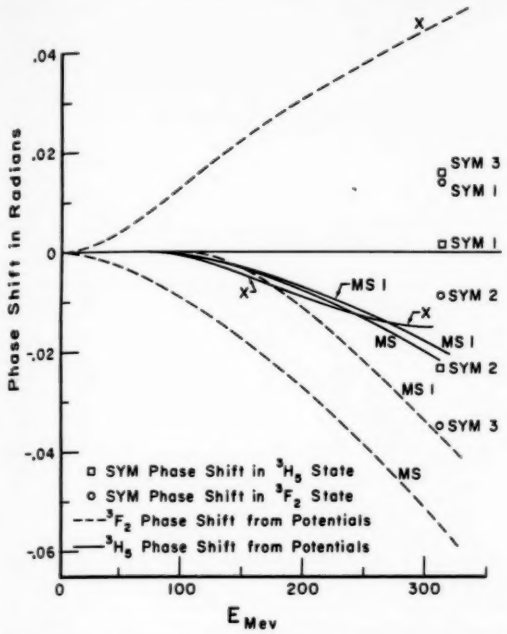

FIG. 1. Values of phase shifts in $^3F_2$ state (dashed curves) and $^3H_5$ state (solid curves) as a function of energy calculated from MS, MS 1, and X potentials. For comparison, the SYM values of the $^3F_2$ phase shift (circles) and $^3H_5$ phase shift (squares) are plotted at 312 Mev for their three best fits, labeled SYM 1,2,3.

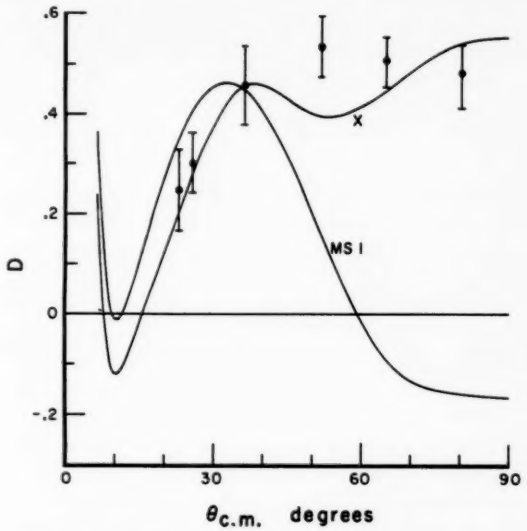

FIG. 2. D vs $\theta_{c.m.}$ at 312 Mev. The data are from Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. 105, 288 (1957). The curves represent values calculated from the MS 1 and X potentials.

referred to as MS. Graphs employing a longer range[4] (16%) of the central singlet-even potential with the spin-orbit potential used only in P states[5] are shown and labeled by X. The three best fits of SYM are referred to by 1,2,3 starting with the best. The combination of F_2 and H_5 is not in agreement with SYM 1,2 or 3 and the claimed improvement in going from MS to MS 1 regarding agreement with SYM 1 does not lead to natural association of the SYM 1 point with the theoretical curves. For some other phase shifts the agreement with SYM 1 is much better, notably so for $^1S_0$, $^3P_{0,1}$, $^1D_2$. For some of these the X potential gives the better agreement.

At 120 Mev, MS 1 and X bracket the curve of polarization P versus center-of-mass angle θ for θ=50° to 90°, MS 1 is the better at 40°, and X is the better from 20° to 30°. However, for the cross section σ the MS 1 values are too high by several times the experimental error while X represents the data appreciably better. At 142 Mev there are disagreements with data outside experimental limits with both versions of the potential for σ, while $P\sigma$ is represented well by both. The depolarization D is apparently represented better by MS 1 than by X. At 240 Mev and $80<\theta<90°$, MS 1 differs from experiment for σ by many times the statistical error while X does not disagree with experiment as badly; at intermediate angles the agreement is better for both MS 1 and X, while for $\theta<20°$ they agree with each other but disagree with experiment. A more complete account of the comparisons at this and other energies is being prepared for publication. There are many instances of radical departure from experiment both below and above 150 Mev. An example of a better fit by means of X to D at 312 Mev is shown in Fig. 2. In view of discrepancies between X and measurements in other respects, no special claims for X are made. It is rather desired to emphasize that both potentials have many deficiencies. A suitable velocity dependence of the potential can produce agreement with experiment but, without restrictions based on theoretical plausibility, such a fit has little significance.

A grant of IBM-704 machine time by the International Business Machine Corporation is gratefully acknowledged.

*This research was supported by the U. S. Atomic Energy Commission and by the Office of Ordnance Re-

search, U. S. Army.

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[2]P. S. Signell and R. E. Marshak, Phys. Rev. 109, 1229 (1958).

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[4]Fischer, Pyatt, Hull, and Breit, Bull. Am. Phys. Soc. Ser. II, 3, 183 (1958).

[5]By introducing the orbital angular momentum operator explicitly in the interaction Hamiltonian, one can arrange for the vanishing of all but the $L=1$ interaction effects. In a certain sense, therefore, the curves X refer to a "potential."

EVIDENCE FOR THE $\Delta I=\frac{1}{2}$ RULE*

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(Received February 16, 1959)

It has been clear for some time that the experimental data relating to strange-particle decays are suggestively close to the values predicted by the $\Delta I=\frac{1}{2}$ selection rule.[1,2] The largest discrepancy has been between the limits[3,2]

$$0.28 \leq B \leq 0.38, \tag{1}$$

predicted for the neutral branching ratio $B \equiv P(K_1^0 \to \pi^0+\pi^0)/[P(K_1^0 \to \pi^0+\pi^0)+P(K_1^0 \to \pi^+ +\pi^-)]$, and the values $B=0.14\pm0.06$ found in the Columbia propane bubble-chamber experiment,[4] and $B=0.06$ (one event) in the MIT iron-plate cloud-chamber experiment.[5] The prediction (1) is obtained, instead of the prediction $B=1/3$ of the "pure" $\Delta I=\frac{1}{2}$ rule, if one assumes enough $\Delta I=3/2$ to account for the decay $K^+ \to 2\pi$.

The particle-mixture theory of Gell-Mann and Pais predicts, for the fraction f of short-lived (K_1^0) decays,

$$f = K_1^0/(\text{all } K^0) = \tfrac{1}{2}. \tag{2}$$

This result is expected from CPT invariance alone, i.e., CP invariance is not needed.[6] If, as seems likely, the 2π modes constitute practically all the short-lived decays,[4] and $K_2^0 \to 2\pi$ is negligible,[7] Eqs. (1) and (2) can be combined to predict, for $\Delta I=\frac{1}{2}$,

$$0.31 \leq R_K \leq 0.36, \tag{3}$$

where $R_K = P(K_1^0 \to \pi^+ +\pi^-)/(\text{all } K^0)$.

The corresponding experimental results (previous to the present experiment) are $R_K=0.42\pm0.05$ (Columbia),[4] and $R_K=0.46\pm0.08$ (Michigan propane chamber).[8] These are only in fair agreement with the prediction (3), but when combined with Columbia's $B/2=0.07$ give $f=0.49\pm0.08$, in good agreement with the particle-mixture prediction (2).

At the time of the CERN conference, we presented preliminary results, based on 450 decay events observed in our hydrogen bubble chamber, for R_K, and for the Λ branching ratio R_Λ.[8] The $\Delta I=\frac{1}{2}$ rule predicts[9] the value

$$R_\Lambda \equiv P(\Lambda \to p+\pi^-)/(\text{all } \Lambda) = 2/3. \tag{4}$$

Our preliminary values were in good agreement with the predictions (4) and (3). We reported no results on the $K_1^0 \to \pi^0+\pi^0$ mode at that time and thus could not check the predictions (1) and (2).

We have now completed our analysis, and report on a total of 1091 events. Our data show no contradictions with the predictions of the $\Delta I=\frac{1}{2}$ rule. On the contrary they are in remarkable agreement with them.

We have observed 227 double events ($D=227$) in which both the Λ and the K^0 decay within the prescribed "fiducial volume" inside the chamber via their charged modes $\Lambda \to p+\pi^-$ and $K_1^0 \to \pi^+ +\pi^-$. There are 594 events in which only the Λ is observed to decay via its charged mode ($\Lambda=594$), and 270 events in which only the K_1^0 is observed to decay via its charged mode ($K=270$). For each of the 864 single V's, the production and decay dynamics of the observed particle checks with the hypothesis of associated production via either $\pi^- + p \to \Lambda + K^0$ (Λ production) or $\pi^- + p \to \Sigma^0 + K^0$ (Σ^0 production).

The branching ratios R_K and R_Λ, and the "true" number of associated productions, n(true), are given by

$$R_K = [D/(\Lambda+D)]\eta_K,$$

$$R_\Lambda = [D/(K+D)]\eta_\Lambda,$$

and

$$n(\text{true}) = [(\Lambda+D)(K+D)/D]\eta_0,$$

where η_K, η_Λ, and η_0 are escape corrections

which would be unity for an infinite chamber. The η's are calculated by averaging over smoothed "true" distributions in the production position and angle. We require that for a Λ or K^0 to be "detectable" its charged decay must occur beyond 0.3 cm from the production point and must lie within the fiducial volume. The fiducial volume is defined by the requirement that all decay tracks be at least 2 cm in length. In the small fraction of single V's in which the production and decay dynamics of Λ and K^0 overlap when curvature and angle measurements alone are used, we measure ionization on the positive decay fragment, to distinguish protons (Λ decay) from π^+ (K_1^0 decay).

The calculated averages of the η's vary by only a few percent over the entire incident-pion momentum range, and differ by only a few percent between the Λ and Σ^0 production modes. Therefore we quote here only the "grand average" over all the "true" distributions, which yields the calculated values $\eta_K = 1.249$, $\eta_\Lambda = 1.398$, and $\eta_0 = 1.097$. In performing the averages we use our lifetime values $\tau_K = 0.94 \pm 0.05 \times 10^{-10}$ sec and $\tau_\Lambda = 2.72 \pm 0.16 \times 10^{-10}$ sec. We also calculate the derivatives with respect to lifetimes, with the results $\Delta \ln R_K = +0.193 \Delta \ln \tau_\Lambda$ and $\Delta \ln R_\Lambda = +0.149 \Delta \ln \tau_K$. The contributions of uncertainties in lifetimes to the uncertainties in R_K and R_Λ are negligible.

The observed counts, D, Λ, and K must be replaced by D/ϵ_2, Λ/ϵ_1, and K/ϵ_1, where ϵ_2 and ϵ_1 are scanning efficiencies for finding double and single V's. By rescanning 30% of the film we find for "detectable" decays, $\epsilon_2 = 0.995 \pm 0.005$, and $\epsilon_1 = 0.976 \pm 0.008$.

The values of R_K and R_Λ obtained at the various beam momenta, 0.95 and 1.03 Bev/c (below Σ^0 threshold), and 1.09, 1.12, and 1.23 Bev/c (above Σ^0 threshold) agree within the errors. Similarly the momentum-averaged Λ-production results agree within one-third standard deviation with the Σ^0-production results. We therefore present only the grand average results, for the fractions of charged decays, and total number of associated productions,

$$R_K = 0.339 \pm 0.020, \tag{5}$$

$$R_\Lambda = 0.627 \pm 0.031, \tag{6}$$

$$n(\text{true}) = 2020 \pm 100. \tag{7}$$

We turn now to the decay $K_1^0 \to \pi^0 + \pi^0$. We have seen three events consistent with this decay mode; in each case there is an associated charged Λ decay. In one case a π^0 undergoes a "Dalitz decay" into $e^+ + e^- + \gamma$. In this other two cases one of the π^0-γ rays produces an electron pair in the liquid hydrogen. From our experimental K^0 momentum distribution we find the π^0-γ-ray spectrum by assuming isotropy in the K^0 and π^0 decay. Combining this with the known pair-production cross section per hydrogen atom, and including a probability for Dalitz-decay of 2.5×10^{-2} per K_1^0, we find a total detection efficiency per K_1^0 of 3.5×10^{-2}. Our three events thus correspond to $3/3.5 \times 10^{-2} = 86 \pm 50$ decays. The number of accidental counts due to chance coincidences from unassociated electron pairs is estimated from the frequency of pairs, and the chance of fitting the decay dynamics. The result is that less than 0.2 accidental count is expected. (No correction was made.) There were 227 decays $K_1^0 \to \pi^+ + \pi^-$, associated with charged Λ decays. Therefore, independent of assumptions as to the value of $f = K_1^0/(\text{all } K^0)$, and independent of escape corrections, we find for the fraction B of neutral K_1^0 decays

$$B = 86/(86 + 227) = 0.27 \pm 0.11, \tag{8}$$

which is consistent with prediction (1). We combine the charged and neutral results (5) and (8) to obtain

$$f = 0.47 \pm 0.080, \tag{9}$$

in good agreement with the particle-mixture-theory prediction (2).

The disagreement between our results (5), (8), and (9) and those of other groups may be expressed as follows. Both Eisler et al.[4] and we find the "expected" $f = \frac{1}{2}$, but disagree on the admixtures of charged and neutral K_1^0 decays. However, if we combine all experiments without imposing the constraint $f = \frac{1}{2}$ then our charged K_1^0 decays dominate statistically, whereas in the neutral decays the Columbia and MIT results prevail (7 + 1 = 8 events to our 3 events). The internal consistency is furthermore not bad – a χ^2 probability of 18% for the charged decays, and 20% for the neutral. The resulting charged and neutral world averages are R_K(U.C., Col.,[4] Mich.[6]) $= 0.354 \pm 0.018$, and B(U.C., Col.,[4] MIT[5]) $= 0.141 \pm 0.037$. (The error on B reflects only the counting statistics.) In the absence of other decay modes of the (short-lived) K_1^0 these combine to give $f = 0.424 \pm 0.026$, in disagreement with prediction (2). If, instead, $f = \frac{1}{2}$ is assumed, then 18 ± 6% of the K_1^0 decays would have to occur

by an as yet unknown mode. We believe that it is instead more reasonable not to combir the experiments, and to attribute the disagreement to statistical bad luck or systematic error. The χ^2 probability that our charged fraction is consistent with the Columbia-MIT neutral fraction is then about 10^{-3}.

We next consider the decay $\Lambda \to n + \pi^0$. We have found two Dalitz decays and one γ conversion corresponding, presumably, to this mode. In each case there was an associated charged K_1^0 decay. The three events correspond to 171 ± 100 neutral Λ decays. Combining these with the 227 double V's, we find $R_\Lambda = 227/(227+171) = 0.57 \pm 0.14$. Since we have good reason for believing that the associated-production hypothesis is valid and that there are no prominent Λ decay modes other than the two considered here, this last result can be combined with result (6) for the charged decays. We thus obtain the weighted average

$$R_\Lambda = 0.624 \pm 0.030. \qquad (10)$$

Result (10) is in excellent agreement with the $\Delta I = \frac{1}{2}$ prediction $R = 2/3$, and with other determinations by groups at Columbia,[4-8] Michigan,[8] and MIT,[5-8] as well as with results of the Berkeley K^--capture experiment.[8] A least-squares weighted average of these results gives R_Λ(U.C., Col., Mich., MIT) $= 0.637 \pm 0.020$.

Finally, Dalitz[1] and Pais and Treiman[10] point out that if the $\Delta I = \frac{1}{2}$ rule is valid one expects $w(K_2^0 \to \pi^+ + \pi^- + \pi^0) = \frac{2}{5} w(K^+ \to 3\pi)$. Furthermore, the decay rate $w(K_1^0 \to \pi^+ + \pi^- + \pi^0)$ should be exceedingly small compared with the corresponding K_2^0 rate,[10] so that we may attribute any observed $\pi^+ + \pi^- + \pi^0$ decay to the K_2^0.

We have seen one such decay. The event was associated with a charged Λ decay. However, we would easily detect this decay mode if it occurred as a single V. Corresponding to the total number of associated productions, $n(\text{true}) = 2020$, there should be 1010 K_2^0's. Combining the known K^+ branching ratios[11] and lifetimes[12] and the $\Delta I = \frac{1}{2}$ rule, one obtains the prediction $w(K_2^0 \to \pi^+ + \pi^- + \pi^0) = 2.4 \times 10^6$ sec^{-1}. The average potential time for K^0's was 3.5×10^{-10} sec. We therefore expect to find $2.4 \times 10^6 \times 3.5 \times 10^{-10} \times 1010 = 0.85 \pm 0.10$ decays $K_2^0 \to \pi^+ + \pi^- + \pi^0$. Our single event is thus consistent with the $\Delta I = \frac{1}{2}$ rule.[13]

In summary, we find that our 1091 associated-production and decay events are in remarkably good agreement with the predictions of the $\Delta I = \frac{1}{2}$ rule.

This makes the total experimental evidence for $\Delta I = \frac{1}{2}$ fairly impressive: (a) The decay $\Xi^- \to n + \pi^-$ occurs, if at all, much less frequently than $\Xi^- \to \Lambda + \pi^-$. This can be understood if the Ξ has $I = \frac{1}{2}$ and the $\Delta I = \frac{1}{2}$ rule holds. (b) The strong inhibition of $K^+ \to 2\pi$ relative to $K_1^0 \to 2\pi$ follows from the $\Delta I = \frac{1}{2}$ rule, if the K has zero spin and $I = \frac{1}{2}$. (c) The admixture of $\Delta I = \frac{3}{2}$ required to admit the observed $K^+ \to 2\pi$ rate is in good agreement with our $K_1^0 \to 2\pi$ branching ratios. (d) The Λ branching ratio agrees with (but does not require[9]) $\Delta I = \frac{1}{2}$. (e) The branching ratio[11,2] $P(K^+ \to \pi^+ + \pi^- + \pi^0)/P(K^+ \to \pi^+ + 2\pi^0)$ agrees with (but does not require[14]) $\Delta I = \frac{1}{2}$. (f) The results of Cool et al.[15] on $\Sigma^\pm$ decay asymmetry are most easily explained by (but do not require) $\Delta I = \frac{1}{2}$. (g) Our one $K_2^0 \to \pi^+ + \pi^- + \pi^0$ agrees (as well as one event can) with $\Delta I = \frac{1}{2}$.[13]

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[13]Bardon et al., reference 7, find that out of 152 K_2^0 decays, at most 23 can be into $\pi^+ + \pi^- + \pi^0$, and also find $\tau(K_2^0) = (8.1_{-2.4}^{+3.2}) \times 10^{-8}$ sec, so that $w(K_2^0 \to \pi^+ + \pi^- + \pi^0) \leq (1.8^{+0.8}_{-0.5}) \times 10^6$ sec^{-1}. Within the errors, this result is consistent with ours, and with $\Delta I = \frac{1}{2}$.
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CHARGE INDEPENDENCE IN THE REACTIONS $p+d \to \pi^0 + \mathrm{He}^3$ AND $p+d \to \pi^+ + \mathrm{H}^3$ AT 450 Mev*

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According to the principle of charge independence, the ratios of the cross sections for the two reactions $p+d \to \pi^+ + \mathrm{H}^3$ and $p+d \to \pi^0 + \mathrm{He}^3$ should be exactly a factor of two. This statement should be true both for the total cross section and the differential cross section. A previous attempt to establish this ratio has been made[1] in which measurements of E and dE/dx were used to identify the He^3. Owing to the large background of protons and deuterons, this separation was difficult to achieve. In order to measure the cross section for the second reaction, both the π^+ and the H^3 in coincidence were detected.

In the present experiment the differential cross sections for these two reactions were measured by counting only the heavy particles. First of all a momentum analysis was performed in order to substantially reduce the background. This momentum analysis was performed with the aid of a magnet which has been previously described.[2] The deflected particles were counted with a quadruple coincidence arrangement consisting of three counters in coincidence and the last one in anticoincidence. An absorber was placed between the last of the three counters and the anticoincidence counter. Upon the receipt of a signal showing that a particle had stopped in the absorber the pulse height in the third counter was measured with a 50-channel pulse-height analyzer. This measurement of pulse height served to distinguish the H^3 and He^3 from the background of protons and deuterons.

The external proton beam from the University of Chicago's synchrocyclotron was focused into a spot about 1/2 in. in diameter on a polymerized CD_2 target. The thickness of this target was chosen to be approximately 0.8 mm. This was necessary because the range of the lowest momentum He^3 particle is only 4 mm of plastic scintillator. The secondary particles from the target were then deflected by the magnet into the counter telescope. The whole system of proton beam pipe, target, and magnet was maintained under vacuum at a pressure of a few microns. The experiment consists of taking CD_2-C differences at various magnet current settings and measuring the pulse-height spectrum at each setting. The magnet can approach within 8 degrees of the forward direction without the proton beam striking the magnet, and the maximum angle of emission of either of the heavy particles is 15 degrees. We have measured the production of these heavy particles from 8 to 13 degrees in the laboratory system for the low-momentum portion of the spectrum. Owing to the small range of the He^3 particles, it was necessary to

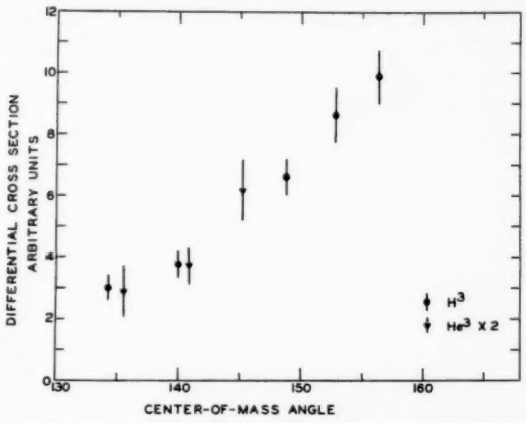

FIG. 1. The differential cross sections for the reactions $p+d \to \pi^+ + \mathrm{H}^3$ and $p+d \to \pi^0 + \mathrm{He}^3$ with 450-Mev protons. The measured cross sections for the second reaction have been multiplied by a factor of two in order to facilitate a direct comparison. The cross sections are given in arbitrary units which are, however, approximately microbarns/steradian.

use only three counters in this part of the experiment. However, measurements on the H^3 were repeated in order to show that the efficiency of detection was identical in the two cases.

The differential cross section of these two processes at the angles measured is shown in arbitrary units in Fig. 1. The differential cross section for the production of He^3 has been multiplied by a factor of 2 on this graph. A weighted mean for the branching ratio $\sigma_{H^3}/\sigma_{He^3}$ has been computed from these data to be 1.91 ± 0.25. Accordingly we conclude that there is no evidence to show that the principle of charge independence does not hold in these reactions.

The authors would like to acknowledge the considerable help given by Charles Rey in preparing for and carrying out these measurements.

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ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

STABLE ORBITS OF CHARGED PARTICLES IN AN OSCILLATING ELECTROMAGNETIC FIELD. Erich S. Weibel, Physical Research Laboratory, Space Technology Laboratories, Incorporated, Division of The Ramo-Wooldridge Corporation, Los Angeles, California (Received November 3, 1958).

Orbits in a field of the cylindrical wave guide driven in the TE_{01} mode have been studied. The radial components of the Lorentz force acting on the particle is never positive regardless of its charge; thus, it is attracted toward the axis of the wave guide around which it moves in a complicated path. Sufficient conditions are given under which the orbits are stable, that is, remain bounded for all times.

NATURE AND ROLE OF IONIZING POTENTIAL SPACE WAVES IN GLOW-TO-ARC TRANSITIONS. Russell George Westberg, The Department of Physics, The University of California, Berkeley, California (Received July 7, 1958).

The transition from a glow discharge to a transient arc, induced in Al-cathode tubes 145 cm long by shorting of a series resistor, has been studied for a variety of gases by observing currents at the electrodes, the potentials at various points in the tube using probes, and the movement of luminous pulses using photomultiplier tubes and fast oscilloscopes. With a dc power supply with up to 2600 volts applied and a large capacity parallel to the discharge tube, and with due regard to proper impedance matching, relatively pure gases N_2, A, H_2, 0_2, and mixtures of N_2 and 0_2 were studied from 50 microns of Hg up to several hundred microns. In addition, shorter tubes with Cu, Ni, Pt, Hg, and W cathodes and filled with He or N_2, at intermediate pressures up to 7.2 mm of Hg, were briefly studied. The decrease of the series resistance increased the very low current in the normal glow mode to higher values up to 50 ma usually in the abnormal mode. With slightly oxidized cathodes, there followed, after times varying from 10^{-3} second to many seconds, catastrophic breakdowns to a transient power arc of the order of 10-20 or more amperes. The transition was initiated by a burst of electrons from the cathode yielding a cathode current increase to several amperes in $2\text{-}10\times10^{-9}$ second. This burst sends a luminous pip or pulse to the anode at velocities varying from 5×10^8 to 8×10^9 cm/sec, depending on pressure, voltage, and gas. On arrival at the anode, a luminous return arc plasma front moves towards the cathode with speeds from 10^8 to 10^9 or more cm/sec. When it arrives at the cathode, the conductivity is sufficiently homogenized along the column so that the current and luminosity increase along the whole column until the drain on the capacity lowers the supply voltage. The luminous pip and, to a less clearly defined degree, the return plasma front are accompanied by potential space waves which can be established by cross-plots yielding luminosity and potential distributions across the tube at various times. The transition appears to be initiated by a sudden breakdown of the oxide layer on the cathode as it thins by ion bombardment, causing a small fraction of the oxide molecules to be ionized within millimicroseconds, thus yielding the initial current burst. These electrons, accelerated by nearly the full potential drop across the dark space, move as ionizing potential fronts, accompanied by luminosity, with velocities in most cases dictated by the cathode fall potential. In O_2, when pressures permit effective photoionization within the tube length, the velocities of the front are increased up to 8×10^9 cm/sec.

AC-DC ELECTROLUMINESCENCE. W. A. Thornton, Research Department, Westinghouse Electric Corporation, Bloomfield, New Jersey (Received April 24, 1958; revised manuscript received August 27, 1958).

The enhancement of electroluminescence, by the superposition of ac and dc voltages, has been observed in certain electroluminescent zinc sulfide powder phosphors. The addition of a dc voltage to an ac voltage exciting visible electroluminescence increases the emission as much as 250 times, under conditions where the dc electroluminescence alone is about equal to the initial ac electroluminescence. Wave-form,

transient, and spectral measurements show that the increased emission is recombination, at dc-ionized luminescence centers, forced by the ac field.

MAGNETIC FIELD DEPENDENCE OF THE SURFACE IMPEDANCE OF SUPERCONDUCTING TIN. M. Spiewak, Department of Physics and Institute for the Study of Metals, University of Chicago, Chicago, Illinois (Received October 27, 1958).

The surface impedance at 1 kMc/sec of superconducting Sn has been investigated as a function of temperature, T, and of static magnetic field, H, both longitudinal and transverse to the rf current. The following unexpected phenomena have been observed. The rf resistance, R, and the reactance, X, may each decrease with increasing H. A decreasing X and an increasing R may coexist. A variation of either the crystalline or static field orientation with respect to the rf magnetic field can change the sign of $[R(H)-R(0)]$ and of $[X(H)-X(0)]$. For $T<2.0°K$, both R and X are monotonically increasing functions of H. In this range of T, the approximation of a quadratic dependence for R on longitudinal and transverse H becomes increasingly unsatisfactory with decreasing T. The quadratic coefficient of the variation of X with longitudinal H also decreases with decreasing T, but the corresponding coefficient in transverse H increases, $1.2°K<T<2.0°K$. The relation between these unexpected behaviors and previous experimental and theoretical studies is discussed.

EXCITATION SPECTRA AND TEMPERATURE DEPENDENCE OF THE LUMINESCENCE OF ZnS SINGLE CRYSTALS. A. Halperin and H. Arbell, Department of Physics, The Hebrew University, Jerusalem, Israel (Received September 3, 1958).

The luminescence of ZnS:Cl and ZnS:Cu:Cl crystals was measured for the temperature region 80-500°K and for different wavelengths of exciting light. The behavior of the luminescence versus temperature curves differed from similar curves for powders reported in the literature.

Excitation spectra were also obtained for the green and blue luminescence bands. These were found to consist of a complexity of fine bands. Each of the bands was found not to shift when varying the temperature of the crystal. The relative intensity of the bands near the absorption edge of the crystal, however, changed with temperature in such a way as to cause an apparent shift in the maximum of the excitation spectrum in this region.

"THIN-FILM" EXPERIMENT WITH BULK SUPERCONDUCTORS. Hans Meissner and Richard Zdanis, The Johns Hopkins University, Baltimore, Maryland (Received October 9, 1958).

Hollow cylindrical samples of indium have been prepared by an extrusion process and equipped with both a center wire and a concentric tube. The critical current has been determined for the case in which the current passes through the sample and returns through the concentric tube. It has been found that this critical current is identical with that of solid samples. The same determination has been made for the case in which the current returns through the center wire. In this case the critical current is about 80 to 90% of that for solid samples. A field was produced by the center wire, the respective current returning through the concentric tube. The resistance of the sample was measured with a small measuring current, also returned through the concentric tube, as a function of the field produced by the center wire. The critical field thus determined depends on the value of the measuring current. These findings can be interpreted in the following way: If the magnetic field of the current through the center wire is dominating, the current in the sample will flow in a very thin layer at the outer surface of the sample. This constitutes a "thin-film experiment" which does not require a thin metal film. The resistance measurements have been supplemented by measurements of the circular flux on a sample of larger size.

EFFECT OF PRESSURE ON THE INFRARED ABSORPTION OF SEMICONDUCTORS. L. J. Neuringer,* Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania (Received May 26, 1958; revised manuscript received December 1, 1958).

Measurements have been made of the effect of hydrostatic pressure upon the intrinsic infrared absorption of germanium, silicon, and tellurium in the pressure range 1-2000 atmospheres. In

germanium the variation with pressure of the lowest lying conduction band minimum, (111), is found to be $(7.3 \pm 1.5) \times 10^{-6}$ ev/atmos. It is suggested that the variation with pressure of the next higher conduction band edge, (000), has a value of $(11 \pm 3) \times 10^{-6}$ ev/atmos. The shift of the infrared absorption edge with pressure in silicon is small and is toward smaller energy, amounting to about -2×10^{-6} ev/atmos between 1 and 2020 atmospheres. The sign and magnitude of the pressure coefficient in this material are in agreement with the results of Warschauer, Paul, and Brooks. For tellurium both $E_{g\perp}$, the energy gap corresponding to light polarized perpendicular to the c axis of the crystal, and $E_{g\parallel}$, the energy gap for polarization parallel to the c axis, decrease as the pressure is increased. It appears that $E_{g\perp}$ decreases more rapidly with pressure than $E_{g\parallel}$. The mean pressure coefficient of $E_{g\perp}$ is $-(2.2 \pm 0.4) \times 10^{-5}$ ev/atmos. For $E_{g\parallel}$ the mean pressure coefficient is $-(1.8 \pm 0.3) \times 10^{-6}$ ev/atmos. The measured pressure coefficients are used to calculate the thermal dilation term in the equation for the change of the energy gap with temperature for each material. The electron-lattice interaction term appearing in this equation is then deduced. In silicon these two terms are of opposite sign with the electron-lattice term dominant. In germanium the electron-lattice interaction effect accounts for 75 percent of the effect of temperature on the energy gap. For tellurium the two effects are almost equal in magnitude but of opposite sign.

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THEORY OF ULTRASONIC CYCLOTRON RESONANCE IN METALS AT LOW TEMPERATURES. T. Kjeldaas, Jr., Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received August 25, 1958).

The extent to which measurement of the propagation characteristics of circularly polarized acoustic shear waves as a function of a magnetic field applied parallel to the direction of propagation can reveal information about the band structure of metals is investigated theoretically. The interaction between the electrons and sound waves is treated classically and is viewed as taking place via an internal electric field and by scattering modified by the sound wave. The treatment is based on the combined solution of Boltzmann's and Maxwell's equations for a free-electron model. A simple physical picture involving cyclotron resonance is developed by means of which the treatment is extended to more general band models. The most striking results are obtained in case the mean free path is larger than the wavelength. Under these conditions, it is found that subject to certain restrictions, (1) absorption edges exist, the measurement of which can be related to the curvature of the Fermi surface; (2) the shape of the Fermi surface may be determinable from the shape of attenuation and dispersion curves within the absorption region; and (3) the cyclotron frequency may be measured. The results (2) and (3) may be dependent upon the simplifying assumption regarding the electron-sound wave interaction.

ROLE OF DOUBLE-QUANTUM TRANSITIONS IN MASERS. Shaul Yatsiv, Stanford University, Stanford, California (Received July 7, 1958).

A generalized treatment of a three-level maser as an example of double-irradiation experiments is based on a previous analysis of multiple-quantum transitions in nuclear magnetic resonance. The emphasis in the present treatment, as distinct from other expositions of the same problem, is on the role of the double-quantum transitions. In particular, conditions are found in which the maser operation is mainly governed by the double-quantum process and does not require a true "pumping" stage. Such a case is realizable in practice but its technical applicability is questionable. A study of the behavior of various transitions in this example is instructive for understanding the general role of the double-quantum transitions in double-resonance experiments.

NUCLEAR STRUCTURE EFFECTS IN INTERNAL CONVERSION COEFFICIENTS BY CONFIGURATION MIXING. L. S. Kisslinger, Western Reserve University, Cleveland, Ohio, and Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received June 2, 1958).

The nuclear matrix elements which are needed to determine the internal conversion coefficients when a finite nucleus is employed are derived for nuclei for which the shell-model wave functions

are a good zero-order approximation for low-energy processes. Using configuration mixing, general expressions are derived for these matrix elements. It is shown that the nuclear structure alteration can be ten to twenty percent or more for l-forbidden transitions. Numerical results are given for the $M1$ and $E2$ 279-kev transitions in Tl^{203}.

SEARCH FOR POSITRON EMISSION IN K^{40}. D. R. Tilley and Leon Madansky, Johns Hopkins University, Baltimore, Maryland (Received May 19, 1958).

A search for positron emission in K^{40} has been made with the use of a triple coincidence technique. An upper limit of $(3.6 \pm 1.8) \times 10^{-4}$ positron per second per gram of natural potassium has been set. A corresponding upper limit of 0.57 ± 0.29 has been computed for the ratio of the squares of the matrix elements, M_+^2/M_-^2, for the K^{40} β^+ and β^- transitions.

ANALYSIS OF THE REACTION $C^{13}(He^3, \alpha)C^{12}$. George E. Owen, L. Madansky, and Steve Edwards, Jr., Department of Physics, The Johns Hopkins University, Baltimore, Maryland (Received July 22, 1958).

The angular distributions of the alpha particles from the reaction $C^{13}(He^3, \alpha)C^{12}$ at 4.5 and 2.0 Mev have been analyzed from the point of view of nuclear stripping. By symmetrizing the wave function of the complete Hamiltonian with respect to the exchange of alpha particles, the heavy-particle stripping amplitude is formally included. The experimental distributions change decidedly between 2.0 and 4.5 Mev, and the calculated curves employing a combination of pick-up and α-stripping from C^{13} agree quite well with the experimental distributions. Interference between the two channels plays a major role in the final expressions.

QUANTUM-MECHANICAL THREE-BODY PROBLEM. Leonard Eyges, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts (Received November 3, 1958).

We treat the quantum-mechanical problem of three spinless particles, with the boundary condition that the logarithmic derivative of the wave function be a prescribed constant at each of the three boundaries $|\vec{r}_1 - \vec{r}_2| = a$, $|\vec{r}_2 - \vec{r}_3| = a$, $|\vec{r}_1 - \vec{r}_3| = a$. This boundary condition is discussed; it is roughly equivalent to an interparticle potential which consists of a hard core plus a strong short-range attractive part. The eigenfunctions and eigenvalues of the system are given by the solutions of an infinite set of coupled homogeneous integral equations. The equations involve partial wave expansions in the interparticle distances and can often be truncated with good approximation by taking only a finite number of partial waves. We discuss the solution of these equations for the ground state of the system, taking relative S-waves only, for which case the infinite set of equations reduces to a single integral equation in one variable.

ALPHA-DECAY BARRIER PENETRABILITIES WITH AN EXPONENTIAL NUCLEAR POTENTIAL: EVEN-EVEN NUCLEI. John O. Rasmussen, Radiation Laboratory and Department of Chemistry, University of California, Berkeley, California (Received October 27, 1958).

The real potential derived by optical-model analysis of data on elastic scattering of alpha particles is used for calculation of barrier penetrabilities for all known alpha decay groups of even-even nuclei. The barrier penetration factors were calculated by numerical integration in the WKB approximation taking into account centrifugal barrier effects but ignoring noncentral interactions. Using these penetration factors and the experimental alpha half-lives, the reduced level widths δ^2 are calculated. Ratios of δ^2 values for ground and excited-state alpha groups are tabulated as a set of reduced hindrance factors.

RATIO OF ASYMMETRIC TO SYMMETRIC FISSION OF U^{233} AS A FUNCTION OF NEUTRON ENERGY. R. B. Regier, W. H. Burgus, and R. L. Tromp, Phillips Petroleum Company, Atomic Energy Division, Idaho Falls, Idaho (Received October 15, 1958).

Radiochemical measurements of the relative yields of the fission products Mo^{99}, Ag^{111}, Ag^{113}, and Cd^{115} from the low-energy neutron-induced fission of U^{233} have been made at various neutron energies. The energies chosen were thermal, 1.8, 2.3, and 4.7 ev, the latter three corresponding to the peaks of previously reported resonances.

It was found that the ratio of asymmetric to symmetric fission is larger at the 1.8- and 2.3-ev resonances than at thermal energies. At the 4.7-ev resonance, however, this ratio is the same as at thermal energies, to within experimental uncertainties. In addition, it was found that the ratio for epi-cadmium neutrons differed from that for thermal neutrons. The results are consistent with Wheeler's prediction that the ratio of asymmetric to symmetric fission should depend upon the spin state of the fissioning nucleus.

DECAY OF As^{74}. D. J. Horen, W. E. Meyerhof, J. J. Kraushaar,* D. O. Wells, E. Brun,† and J. E. Neighbor,‡ Stanford University, Stanford, California (Received October 13, 1958).

The decay of As^{74} (17.5 day) has been reinvestigated with single and coincidence scintillation spectrometers. The singles spectrum revealed previously unknown gamma rays of 1.19 ± 0.01, 1.60 ± 0.04, and 2.22 ± 0.02 Mev. X-ray–gamma coincidence measurements indicate the 1.19-Mev gamma ray is predominantly associated with the decay to Ge^{74}. Evidence is found for 0.6–0.6-Mev gamma-gamma coincidences in Ge^{74}. Based on these data, a level scheme for Ge^{74} is presented with excited states at 0.596, 1.19, and 2.22 Mev. No new information concerning the decay to Se^{74} has been gained.

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LOW-INTENSITY CONVERSION LINES FROM Sm^{151} AND Sn^{113}. W. T. Achor, Vanderbilt University, Nashville, Tennessee, and Western Reserve University, Cleveland, Ohio, and W. E. Phillips,* J. I. Hopkins,† and S. K. Haynes,‡ Vanderbilt University, Nashville, Tennessee (Received November 13, 1958).

The electron spectrum of Sm^{151} has been measured with a lens spectrometer and its gamma-ray spectrum with a scintillation spectrometer. The L-conversion coefficient of the 21.7 ± 0.3 kev de-excitation of the first excited state of Eu^{151} is 20 ± 4, with $\alpha_L/\alpha_{M+N} = 2.2 \pm 0.4$ and $\alpha_M/\alpha_N = 2.2 \pm 0.2$, indicating $M1$ multipolarity. The low intensity of the conversion electrons relative to the continuous beta spectrum of end point 75.9 ± 0.6 kev implies the existence of a weak beta transition whose end point is 54.2 ± 0.7 kev. The beta branching ratio, 54.2-kev beta transitions/75.9-kev beta transitions, is 1.7/100.

$\text{Log}_{10}ft$ values for the two beta decays are 7.6 ± 0.2 for the 75.9-kev decay and 8.8 ± 0.2 for the 54.2-kev decay. A partial analysis of the decay in terms of the Nilsson-Gottfried scheme has been made, but no definitive assignment for the 21.7-kev state appears to be possible. Measurements of the electron spectrum of Sn^{113} with the lens spectrometer reveal the existence of a weak conversion line corresponding to a transition of 253 kev. The intensity of this conversion line is $(2.8 \pm 1.0) \times 10^{-3}$ of the intensity of the conversion line from the 392-kev transition. This relative conversion electron intensity indicates that the 253-kev transition leads to the 392-kev state of In^{113}, implying the existence of an excited state at 645 kev. The existing uncertainty in the relative gamma-ray intensities for the two transitions makes it impossible to assign spin and parity to the 645-kev state; the decay scheme presented is therefore incomplete.

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$H^3(\alpha,\gamma)Li^7$ AND $He^3(\alpha,\gamma)Be^7$ REACTIONS. H. D. Holmgren and R. L. Johnston, Nucleonics Division, United States Naval Research Laboratory, Washington, D. C. (Received October 14, 1958).

The cross sections for the $H^3(\alpha,\gamma)Li^7$ and $He^3(\alpha,\gamma)Be^7$ reactions have been measured at bombarding energies of 0.48, 0.72, 0.94, 1.13, and 1.32 Mev. The cross sections at the lower bombarding energies may be fitted within the experimental uncertainties by the expressions

$$\sigma = 0.12(1-0.51E_\alpha)E_\alpha^{-1} \exp(-3.96E_\alpha^{-\frac{1}{2}})\ \mu\text{b}$$

for the H^3 reaction and

$$\sigma = 2.8(1-0.55E_\alpha)E_\alpha^{-1} \exp(-7.92E_\alpha^{-\frac{1}{2}})\ \mu\text{b}$$

for the He^3 reaction.

HEAVY-MESON–HYPERON RELATIVE PARITY AND THE SIGN OF THE COULOMB INTERFERENCE IN K^--p SCATTERING. E. Galzenati and

B. Vitale, Istituto di Fisica Teorica dell'Università, Napoli, Italy, and Scuola di Perfezionamento in Fisica Teorica e Nucleare del C.N.R.N., Napoli, Italy (Received October 31, 1958).

Two subtracted dispersion relations are applied at the same time to the study of the dependence of the real part of the K^--p scattering amplitude at zero energy, $D_-(K)$, on the values of the coupling constants g_H^2. The results, when compared with the existing experimental information, strongly indicate that the K meson is ps-ps (namely pseudoscalar with respect to both the Λ and the Σ hyperon) and that the sign of $D_-(\omega)$ at low energy is positive, giving therefore constructive Coulomb interference in the K^--p elastic scattering.

EFFECTIVE MEAN FREE PATH OF ANTINUCLEONS IN NUCLEAR MATTER. Jose R. Fulco, Radiation Laboratory, University of California, Berkeley, California (Received November 6, 1958).

The effective mean free path for antinucleons in nuclear matter is calculated at intermediate energies from the Fermi gas model, in terms of the nucleon-antinucleon total and differential scattering cross sections. The results are used to obtain the imaginary part of the optical-model potential in the formalism of Riesenfeld and Watson.

DECAY OF ${}_\Lambda H^3$ AND THE SPIN DEPENDENCE OF THE Λ-NUCLEON INTERACTION. M. Leon, Laboratory of Nuclear Studies, Cornell University, Ithaca, New York (Received October 24, 1958).

A calculation is made of the fraction of the π^- mesonic decays of the hypertriton ${}_\Lambda H^3$ which yield the two final products He^3 and π^-. This fraction is a function of the spin of ${}_\Lambda H^3$ and of the ratio p/s of the amplitudes for decay of the free Λ via the s- and p-wave channels. The results are compatible with the present experimental data. They indicate that probably $p/s \leq 1$, and that the spin of ${}_\Lambda H^3$ is 1/2, which implies that the singlet Λ-nucleon interaction is more attractive than the triplet. These results are in agreement with those of a previous calculation by Dalitz for ${}_\Lambda H^4$.

REPRESENTATIONS OF THE DIRAC EQUATION. S. K. Bose, A. Gamba, and E. C. G. Sudarshan,* Department of Physics and Astronomy, University of Rochester, Rochester, New York (Received October 20, 1958).

A representation of the Dirac equation which displays its extreme relativistic properties is discussed. The "spin" appears naturally related to the "position."

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MESON-MESON SCATTERING TERM AND STATIC MODEL IN Ps-Ps MESON THEORY. M. Sugawara, Purdue University, Lafayette, Indiana, and A. Kanazawa, Hokkaido University, Sapporo, Japan (Received March 17, 1958).

A static Hamiltonian has been constructed from the renormalizable covariant pion field theory in a manner similar to that recently proposed by one of the present authors. Improvements are that the meson-meson scattering term is included besides the ps-ps coupling term, while the latter alone was assumed previously, and that an argument is presented to show that the Foldy transformation is the unique one generating a valid static Hamiltonian, though it was left undetermined before. The resulting static Hamiltonian has been analyzed, for the cases of low-energy S- and P-wave pion-nucleon scattering and threshold gamma-pion production, in terms of the one-meson approximation of the Chew-Low-Wick formalism without recourse to perturbation expansion. It is found in particular that the meson-meson scattering term modifies the Chew-Low effective range plot of the δ_{33}-phase shift, making the renormalized P-wave coupling constant smaller than the customary plot gives, for a positive coefficient of the meson-meson scattering term in the Hamiltonian. Empirical values of the coupling constant determined through the Chew-Low plot and threshold gamma-pion production are shown to be interpretable in terms of the renormalized P-wave coupling constant of ≈ 0.08 and the meson-meson scattering term with coefficient of $\approx +7$ ($\hbar = c = 1$). The present treatment of threshold gamma-pion production is criticized in comparison with the relativistic dispersion relation. General features of the static model resulting from the ps-ps mes-

on theory are summarized in the final section, together with the conclusions obtained. The effects of strange particles and of renormalization have been neglected.

SOLUTION OF MAXWELL'S EQUATIONS IN TERMS OF A SPINOR NOTATION: DIRECT AND INVERSE PROBLEM. H. E. Moses, Institute of Mathematical Sciences, New York University, New York, New York (Received September 15, 1958).

Maxwell's equations for fields with sources in media in which the dielectric constant and permeability are unity are written in terms of a spinor notation which resembles the one used for Dirac's equation for the electron. One can introduce Green's functions and expansions in terms of complete sets of orthogonal functions, analogous to those used in the quantum theory of the electron, to solve Maxwell's equations in more compact form than in terms of the conventional vector notation.

In addition, the new notation enables us to solve in a simple way an "inverse radiation problem" which we describe as follows: Consider at time $t<0$ the electromagnetic field to be zero. At time $t=0$ sources are turned on and then later turned off. The electromagnetic field, which results after this process has been completed, will be a radiation field. We can solve the problem of finding the nature of the sources which will lead to a prescribed final radiation field. It is shown that, in general, the sources are not unique but additional conditions can be given which will make them so.